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Novel two-node linear composite beam element with both interface slip and shear deformation into consideration: Formulation and validation

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ABSTRACT

This paper develops a novel two-node linear composite beam element for the steel–concrete composite beam with elastic shear connection. The element stiffness matrix takes into account the effects of interface slip and shear deformation. Both simply supported- and continuous composite beams are used to validate the proposed element, and a comparison is made between the proposed element and other methods. The impact of cross sectional shear deformation on deflection is also discussed. The results show that the novel element is capable of effectively and accurately calculating the deflection and slip, and is applicable to slender composite beams or deep ones.

1. Introduction

In the past two decades, composite beams have been widely applied in building floors, bridge decks and other large span structures due to their virtues, such as high strength, high stiffness and good ductility. In the case of simply supported composite beam, the concrete slab is mainly subjected to compressive stresses, while the steel beam is subjected to tensile stresses. In view of composite action between steel beam and concrete slab, shear connectors are arranged at the interfaces of composite beams, which transfer the shear stress from one component to the other thus resulting in composite action. With the use of rigid shear connectors which have infinite stiffness to eliminate the interface slip, a rigid shear connection or full composite action between the concrete slab and the steel beam can be assumed. Unfortunately, it is difficult to achieve a rigid shear connection due to the deformability of shear connectors with finite stiffness. Thus, elastic shear connection or partial shear interaction is observed in most of composite beams in light of the interface slip between the two components of a composite beam. The cases of no shear connection and rigid shear connection are just the two extremes of elastic shear connection. As a matter of fact, interface slip always exists in composite beams, which will decrease composite effects and stiffness of composite beams and increase deflections. In addition, in the case of short and thick layered beams where the span–depth ratio is small, the effect of cross-section shear deformation can be significant; therefore the Euler–Bernoulli beam theory with interface slip can be questioned. The shear deformation will give rise to problems like cross-sections not being normal to the neutral axis after deformation. Consequently, several numerical models and analytical formulations have been proposed and are currently available within the scientific literature.

One of the earliest and most cited of the works on the partial shear interaction of composite beams is that of Newmark et al. [1], but it used the Euler–Bernoulli beam theory with which no shear deformation is allowed for. The following studies [2–7] on composite beams mostly used the Newmark theory directly to analyze composite beams with slip. Ranzi et al. [8] presented a direct stiffness method that was applied to a continuous composite beam. However, in his work, the refined analytical method was based on the Euler–Bernoulli beam theory and the effect of shear deformation was not considered. In addition, several closed-form solutions [9–11] have been proposed. As shown in Zhou et al. [11] the closed-form solutions neglect the shear deformation for the composite beam with interface slip, and it is well known that closed-form solutions vary with different loading conditions and

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constraint conditions. The closed-form solutions can only be obtained for simple cases, such as simply supported beams under simple loads and it is difficult to obtain the closed-form solutions for continuous beams. Moreover, it is too complicated to obtain closed-form solutions in practical applications. As a consequence, it will become harder to obtain closed-form solutions if the effects of interface slip and shear deformation are taken into account simultaneously.

Alternatively, numerical methods are versatile and general in dealing with any combination of loading and boundary conditions. A number of two-dimensional and three-dimensional composite beam elements and models have been successfully developed to analyze the composite beams [12–17]. Although these elements can usually produce relatively more accurate models than one-dimensional elements, they are much more complicated and cost more time than one-dimensional elements. In view of this point, numerous scholars investigated this issue and a number of one-dimensional composite beam elements have been developed for the finite element analysis of composite beams [18–22]. In these studies, one-dimensional elements are all based on higher order theories; furthermore, they need more memory requirements for data storage and more runtime compared with linear elements. Most composite beam element models to date employed classical Euler–Bernoulli beam theory with interface slip proposed by Newmark et al. [1]. It is well known that this theory neglects the shear deformation of the cross-section. Recently, the most significant advances in the theory of composite beams with elastic shear connection moved toward the introduction of shear deformation of both components according to the Timoshenko beam theory. Contributions on this subject have been presented in [23–26]. Ranzi and Zona [23] developed a finite element model, in which the reinforced concrete slab is considered as an Euler–Bernoulli beam and the steel beam is considered as a Timoshenko beam. Xu and Wu [24] developed an analytical model considering the Timoshenko kinematic assumption for each component but they imposed the constraint of equal cross-section rotation on both components. On the basis of the Timoshenko's theory, Martinelli et al. [25] developed a closed-form "exact" one dimensional finite element for shear flexible steel–concrete composite beams in partial interaction, considering the effect of shear flexibility for both concrete slab and steel beam, but they adopted a common rotation for the two components in every section and the continuous bond model for the interface connection. In addition, Nguyen et al. [26] presented an exact finite element matrix for a two-layer Timoshenko beam element with continuous shear connection. It is well known that the two components of composite beam are often connected in a discrete way by means of shear connectors. Therefore, Nguyen et al. [27] presented an exact finite element model for shear-deformable two-layer beams with discrete shear connection, which is modeled using concentrated spring elements at each connector location. The corresponding exact stiffness matrix was deduced by employing the Timoshenko kinematic assumption and the governing differential equations which need to take into account the direction of cross sectional forces and to adopt the higher derivation. In this way, the solution procedure is too complex to be applied to practical engineering. In light of this, the goal of this paper is to develop a one-dimensional linear composite beam element with discrete shear connection, while it is unnecessary to set concentrated spring element at each connector location, and the corresponding element stiffness matrix is deduced by using the energy approach.

In this paper, a novel two-node linear composite beam element is developed by using the finite element method. The key step for the finite element method is to construct the element stiffness matrix, which is derived by using the total potential energy method based on the Timoshenko beam theory (TBT) and the linear Lagrangian interpolation function in this paper. Nevertheless, it is not an easy task to eliminate the slip at the interface of a composite beam in practice; furthermore, the composite beam may be sometimes designed as a deep beam or a thin web steel beam. In this case it is necessary to take into account the effects of interface slip and also that of cross sectional shear deformation when deducing the element stiffness matrix. To validate the accuracy of the proposed composite beam element, the deflections of both simply supported- and continuous composite beams are calculated by using the developed finite element program and the results are compared with those of the existing finite element model [26], as well as with experimental data. After having been validated by the simply supported composite beam and continuous beam, the proposed element is applied to both continuous composite beam and deep composite beam so as to analyze the effects of interface slip and shear deformation on deflection. The results show that the proposed element is efficient in calculating interface slip and deflection of composite beams with the shear deformation also taken into consideration, and the proposed element provides unified formulation for the finite element analysis of slender composite beams or deep ones. Furthermore, it is unnecessary to consider the restriction of loading and constraint conditions.

The rest of the paper is organized as follows. Timoshenko beam theory is reviewed in Section 2, and then the element stiffness matrix is derived for a composite beam based on TBT by using the total potential energy method in Section 3. With the help of the element stiffness matrix, a finite element program is developed to analyze the deflection and interface slip of both simply supported composite beam and continuous beam in Section 4. Section 5 gives some conclusive remarks.

2. Timoshenko beam theory

2.1. Displacement field

In a typical Timoshenko beam [28], as shown in Fig. 1, the cross-section that is usually assumed to be normal to the neutral axis before deformation remains straight but not necessarily normal to the neutral axis after deformation. Thus the displacement $\bar{u}$ at any point (x, z) of cross-section in the longitudinal direction can be expressed directly in terms of $\theta(x)$ the rotation of the normal so that

$$\bar{u}(x, z) = -z\theta(x) \quad (1)$$

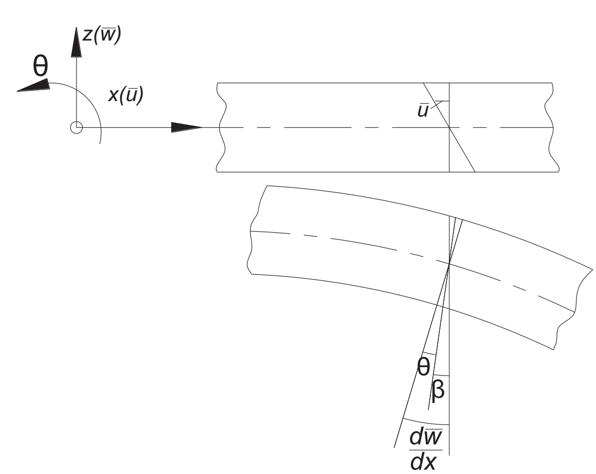

Fig. 1. The Timoshenko beam.