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Size dependent free vibration analysis of multicrystalline nanoplates by considering surface effects as well as interface region

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ABSTRACT

This paper, in line with the previous study [23], is concerned with the finite element implementation of nanoplates. However, in this contribution free vibration responses of multicrystalline nanoplates by considering surface effects are presented. Nanomaterials and nanostructures have been receiving widespread attentions during last decades. This fact is due largely to surprising, peculiar, and impressive mechanical; electrical; and physical behaviors of nanostructures. Currently, nanostructures such as nanoplates are being utilized in the designing and manufacturing Nanoelectromechanical systems (NEMS) and Microelectromechanical systems (MEMS). Furthermore, silicon, thanks to its exceptional mechanical, physical, and electrical properties is extensively employed in the NEMS and MEMS. The mechanical properties and responses of nanoplates are intensely size-dependent, and in contrast to plates with macro dimensions, static and free vibration responses of nanoplates strongly depend on the size of nanoplates. In this study, a rectangular multicrystalline plate with nanothickness; arbitrary geometry, and boundary conditions is analyzed. Each crystal of the nanoplate is assumed to be anisotropic, and a prominent point that must be taken into consideration is the interface region, which exists between every two crystals. The free vibration responses of nanoplate such as natural frequency are considered, and the influence of size, surface effects, interface region, and various boundary conditions over natural frequency of the nanoplate is considered. Due to the fact that geometry of the multicrystalline nanoplate is not straightforward to be dealt with the governing equations, the finite element method is employed to obtain the results of free vibration response. Moreover, we succeed to employ ANSYS software in order to attain the free vibration responses of multicrystalline nanoplates. In addition, the present finite element method results, the code of which is generated in MATLAB, are compared with those obtained from ANSYS software, and the correlation of the results is quite remarkable.

1. Introduction

In contrast to mechanical properties of macrostructures, the mechanical properties of nanostructures such as Modulus of elasticity, Poisson's ratio, natural frequency, to name but a few, are intensely dependent on the size and dimension of the nanostructures. Hence, make predictions about mechanical properties and behaviors of nanostructures that are quite complicated. Due to the fact that the energy of atoms in the bulk of nanostructures differs from that of the surface, surface effects namely surface elasticity (E^s), surface residual stress (τ^S), and surface mass density (ρ^S) appear in the analysis of nanostructures. Numerous investigations have been done in order for the mechanical properties of nanoplates, nanobeams, nanotubes, and nanowire to be attained. The effects of surface and interface stresses on the mechanical behaviors and responses of thin films were studied by Stoney [1]. Zhang et al. studied bending of nanobeams by considering surface effects [2]. Wang and Zhao studied self-buckling and bending behaviors of nanoplates by considering the significance of surface effects [3]. Besides this contribution, a theoretical model is presented by Guo and Zhao so as to determine size-dependent bending properties of nanobeams [4]. In addition, they presented a three- dimensional theory in order to investigate the size-dependent elastic constants of thin films [5]. They introduced not only a parameter to define the ambiguity of nanofilms thickness, but also a relaxation coefficient to modify the lattice constant of surface atoms. Furthermore, by considering the inward or outward surface relaxation effects, the increasing and decreasing elastic modulus with the decrease of size can be predicted.

A simulation on the mechanical behavior of nanostructures and by employing molecular dynamics was presented by Liu and

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coworkers [6]. Wong and coworkers established the mechanical properties such as young's modulus, strength, and toughness of nano rods and nano tubes [7]. Surface effects alter the mechanical properties and behavior of nanostructures. For instance, when the thickness of layers in a nanostructure decreases, the modulus of elasticity may decrease or increase [8–11]. Interface region, which plays a leading role in the analysis of multicrystalline nanostructures, may possess different structures and properties as compared to its neighboring material. Some notable studies have been done on the interface region [12–15].

The development of fundamental theory of elasticity which is used in the analysis of nanostructures was presented by Gurtin and Murdoch [16]. They also presented the effects of surface stress in solids and structures [17]. In contrast to macrostructures, nanostructures may be deformed, although external loads may not exist. This tendency to deformation in the absence of external loads is directly related to surface residual stress. Dunham and Gurtin presented a theoretical theory on the effect of τ^0 on the deformation of crystals [18]. Based on the multiscale dynamics, Choi and coworkers presented a multiscale dynamic model to investigate the dynamic behavior and natural frequency of nano-sized thin films [19]. In addition, the dynamics behavior and natural frequency of buckled thin films by considering surface effects as well as residual surface stress was studied by Wang and Feng [20]. Employing theoretical relations of nanostructures are intensely limited to systems of elementary geometry and boundary conditions. Consequently, it is much more appropriate that alternative approaches such as the finite element method (FEM) to be implemented. Thus this work is presented by employing FEM. Developing the basic and fundamental relations of FEM by considering surface effects was proposed by Gao and coworkers [21]. Tian and Rajapakse presented finite element relations for anisotropic material and for structures with arbitrary geometry [22]. Mouloodi, et al. presented the static responses of multicrystalline nanoplates by considering surface effects [23]. They ascertained a notable procedure in order to analyze nanoplates in ANSYS software. In addition, the deformation of nanostructures due to surface effects, and by implementing the finite element method was studied by Javili and Steinmann [24–25]. So as to attain the modal responses of the multicrystalline nanoplate, elementary relations are derived, and FEM code is generated. Ultimately, the multicrystalline nanoplate is simulated in ANSYS software. In the analysis, it is assumed that the multicrystalline nanoplate is made of silicon, which is a prevalent material in designing nanostructures. The values of silicon surface effects, the mechanical properties of silicon, and its amorphous properties have been studied by numerous researchers [26–35]. In this contribution, a nanoplate which is consisted of several crystals with different mechanical properties and various surface effects is taken into account. Owing to the presumption that the nanoplate is made of silicon, the interface region is the amorphous of silicon. The natural frequency of multicrystalline nanoplate is presented. Moreover, the effects of nanoplate thickness; positive and negative surface elasticity; and the effect of amorphous width on the values of natural frequency are portrayed.

2. Fundamental and governing equations

In order to obtain the free vibration responses of nanoplate, a multicrystalline nanoplate of nanothickness is assumed, as shown in Fig. 1. Each crystal of the nanoplate is assumed to be orthotropic, and with respect to the mechanical properties of silicon, the natural frequency of multicrystalline nanoplate is presented.

As it is mentioned, in designing as well as analyzing multicrystalline plates, existence of interface region is indisputable. In this study the interface region is presumed to be the amorphous of crystal materials. Due to the fact that the crystals are assumed to be made of silicon, therefore, interface region is the amorphous of silicon. Surface effects are prominent factors in the analyzing multicrystalline nanoplate due to the high ratio of surface to volume of the nanoplate. Accordingly, the fundamental equations of surface must be taken into consideration along with the fundamental equations of bulk.

2.1. Constitutive relations

It is assumed that the surface covers top and bottom of the multicrystalline nanoplate, are shown in Fig. 2. In order to consider the behavior of bulk materials, the classical theory of continuum mechanics is employed

$$[\sigma] = [c][\varepsilon] \tag{1}$$

where, $[\sigma]$ and $[\varepsilon]$ demonstrate the stress and strain matrix, respectively, and [c] demonstrates the stiffness tensor for the anisotropic case.

As previously mentioned, it is presumed that the case of orthotropic occurs for the nanoplate. In addition, plane stress case is employed for nanoplates. Regarding these presumptions, Eq. (1) can be expressed as

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix} = \begin{bmatrix}\frac{E_x}{1-\upsilon_{xy}\upsilon_{yx}} & \frac{\upsilon_{xy}E_x}{1-\upsilon_{xy}\upsilon_{yx}} & 0\\ \frac{\upsilon_{xy}E_x}{1-\upsilon_{xy}\upsilon_{yx}} & \frac{E_y}{1-\upsilon_{xy}\upsilon_{yx}} & 0\\ 0 & 0 & G_{xy}\end{bmatrix}\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix} \tag{2}$$

Therefore, the stiffness matrix of bulk material, which is used in the finite element relations, can be derived as

$$\sigma = [D^b]\varepsilon \tag{3a}$$

$$[D^b] = \begin{bmatrix}\frac{E_x}{1-\upsilon_{xy}\upsilon_{yx}} & \frac{\upsilon_{xy}E_x}{1-\upsilon_{xy}\upsilon_{yx}} & 0\\ \frac{\upsilon_{xy}E_x}{1-\upsilon_{xy}\upsilon_{yx}} & \frac{\upsilon_{xy}E_y}{1-\upsilon_{xy}\upsilon_{yx}} & 0\\ 0 & 0 & G_{xy}\end{bmatrix} \tag{3b}$$

In order to consider surface effects, which plays a leading role in analyzing nanoplates, a stiffness matrix for the surface of the nanoplate must be derived. Gurtin and Murdoch [16] presented the linear stress–strain equation for the surface of nanostructures, which definitely can be implemented for multicrystalline nanoplates,

$$\sigma^s_{\alpha\beta} = \tau^0\delta_{\alpha\beta} + 2(\mu^s - \tau^0)\varepsilon_{\alpha\beta} + (\lambda^s + \tau^0)\varepsilon_{kk}\delta_{\alpha\beta} \quad (\alpha,\beta = 1,2) \tag{4}$$

where,$\sigma^S_{\alpha\beta}$ and $\varepsilon_{\alpha\beta}$ denote the surface stress and the surface strain, respectively, λ^S and μ^S are the Lame's constants of the surface, τ^0 is the surface residual stress at zero strain. Surface residual stress plays a leading role in the analysis of nanostructures, and its effects on the mechanical behaviors of nanostructures cannot be overlooked. Wang et al. investigated the effects of surface tension on the elastic properties of nanostructures [36]. They investigated the influences of surface tension and surface residual stress, which is induced by surface tension, in the bulk of nanostructures. Moreover, they deemed that surface residual stress is so significant that it cannot be overlooked for certain cases.

A stress–strain relation is attained for the surface of multicrystalline nanoplate, by developing the Eq. (4)

$$\begin{bmatrix}\sigma^s_{xx}\\ \sigma^s_{yy}\\ \sigma^s_{xy}\end{bmatrix} = \begin{bmatrix}2\mu^s+\lambda^s-\tau^0 & \lambda^s+\tau^0 & 0\\ \lambda^s+\tau^0 & 2\mu^s+\lambda^s-\tau^0 & 0\\ 0 & 0 & 2(\mu^s-\tau^0)\end{bmatrix}\begin{bmatrix}\varepsilon^s_{xx}\\ \varepsilon^s_{yy}\\ \varepsilon^s_{xy}\end{bmatrix} + \begin{bmatrix}\tau^0\\ \tau^0\\ 0\end{bmatrix} \tag{5}$$