International Journal of Mechanical Sciences 85 (2014) 168–178

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

On the use of bubble complex finite strip method in the nonlocal buckling and vibration analysis of single-layered graphene sheets

Saeid Sarrami-Foroushani, Mojtaba Azhari *

Department of Civil Engineering, Isfahan University of Technology, Isfahan 84156-83111, Iran

ARTICLE INFO

Article history:
Received 19 September 2013
Received in revised form
6 April 2014
Accepted 24 May 2014
Available online 2 June 2014

Keywords:
Single-layered graphene sheets
Nonlocal elasticity theory
Free vibration
Buckling
Bubble complex finite strip method

ABSTRACT

In the present work, the buckling and vibration of rectangular single-layered graphene sheets is analyzed based on the nonlocal theory of elasticity which takes the small scale effects into account. The graphene sheet is assumed as a thin plate, and the classical plate theory is applied to obtain the differential equation of the sheet. For the first time, the complex finite strip method is employed to study the vibration and buckling behavior of graphene sheets. The weighted residual method is employed to obtain the stiffness, stability and the mass matrices of the graphene sheet which is assumed to be an isotropic nanoplate. A sinusoidal displacement function is used for the longitudinal direction, which satisfies the simply supported boundary condition, while piecewise interpolation polynomials including the Hermitian and bubble functions are assumed for the other direction. A matrix eigenvalue problem is solved to find the vibration frequency and buckling load of graphene sheets subjected to different types of in-plane loadings including the uniform and non-uniform uniaxial and biaxial compressions as well as shear loading. The accuracy of the proposed model is validated by comparing the results with those reported by the available references. Furthermore, a number of examples are presented to investigate the effects of various parameters (e.g., boundary conditions, nonlocal parameter, aspect ratio, and type of loading) on the results.

1. Introduction

The number of studies on nanostructures and their applications has been rapidly increasing in the scientific community during the last decades. Carbon nanotubes (CNTs) and Graphene sheets (GSs) are the two most cited nanostructures. CNTs, which were discovered in 1991 by Iijima [1], have many potential applications due to their great mechanical, chemical, thermal, electrical and electronic characteristics [2]. Graphene, which was obtained by Geim and his colleagues in 2004 [3], is also another famous type of carbon nanomaterial with a two-dimensional structure. It possesses many excellent properties including good flexibility and high thermal and electrical conductivities [4,5]. Because of their unique characteristics, the application of these nanostructures is rapidly growing in different areas [6–8]. Therefore, analyzing and understanding the different behaviors of these nanostructures is very important and necessary for an effective design. For this purpose, different experimental, analytical and numerical methods have recently been implemented. These approaches can be categorized as atomistic-based methods and continuum-based methods. Although the results of atomistic-based methods such as molecular dynamics (MD) simulation [9,10], tight-binding molecular dynamics [11] and density function theory [12] are more realistic, these methods are computationally expensive, as they involve a large system of atoms. On the other hand, the experimental methods are difficult to implement in most cases. These reasons have prompted the researchers to develop appropriate mathematical models for the analysis of nanostructures. The classical continuum mechanics theories have been recently applied to nanostructures in order to develop appropriate mathematical models that can provide rational results. However, experiments do not confirm the results obtained from the classical continuum mechanics models. It has been shown that the behavior and mechanical properties of nano-materials are influenced by scale effect or size effect. Therefore, to achieve accurate results, the small-scale effects must be considered. Different size-dependent continuum theories such as the couple stress elasticity theory [13], strain gradient theory [14], micro-morphic theory [15] and the surface energy incorporated continuum theory [16] have been developed. Recently, the use of the nonlocal elasticity theory (first proposed by Eringen [17–19]) in the analysis of nanostructures has received significant interest. In this theory, the interatomic forces and the internal length scale are considered in the formulation. The small-scale effects are captured by assuming that the stress at a reference point is a function of the strain field at every point in the domain. The results of nonlocal models are in

* Corresponding author. Tel.: +98 31 33913804; fax: +98 31 33912700.
E-mail address: mojtaba@cc.iut.ac.ir (M. Azhari).

http://dx.doi.org/10.1016/j.ijmecsci.2014.05.024
0020-7403/

accordance with the atomistic results of lattice dynamics and MD simulations [20]. A large number of nonlocal model-based studies that focus on the static bending, dynamic vibration and the stability analysis of nano-structures have been reported. These include the analyses of nanobeams [21–24], nanotubes [25–30], nanorods [31], nanorings [32] and nanoplates [33–35].

Graphene sheets, as a group of nanostructures, have been the subject of many research works. Due to the increasing applications of GSs, understanding their mechanical behaviors is essential for proper engineering design and manufacture. In most of the studies, a graphene sheet is considered as a nanoplate. Different vibration and buckling studies of single- and multi-layered graphene sheets with and without the surrounding elastic medium have been reported in the literature [36–61]. These papers have mostly applied the nonlocal elasticity theory and used different methods such as the differential quadrature method (DQM) and the finite element method (FEM).

The present work is focused on studying the buckling and vibration behaviors of single-layered graphene sheets (SLGSs) by means of the bubble complex finite strip method (BCFSM) in which the bubble functions are employed to improve the accuracy and efficiency of the method. In comparison to other numerical methods such as the FEM, the finite strip method (FSM) provides more efficient formulations for the investigation of plate behavior under different loads and boundary conditions. The FSM was first introduced by Cheung [62]. Later, Przemieniecki used this method to predict the initial local buckling stresses of plate assemblies under biaxial compression [63]. Plank and Wittrick employed the complex finite strip method (CFSM) to investigate the buckling of thin, flat-walled structures under combined loading [64]. Azhari and Bradford [65–68] used bubble functions and employed different versions of the finite strip method to study the local buckling and post-local buckling of plates and plate assemblies. Bubble functions are the finite element shape functions that are zero at the boundary of the element, but nonzero at the other points. These functions have been used in order to augment the finite element formulations to achieve faster convergence.

Sarrami and Azhari [69] used the ordinary finite strip method to analyze the vibration and buckling behavior of SLGSs and MLGSs, incorporating the nonlocal effects in the formulation. The results show that in comparison with other nonlocal approaches, e.g. DQM, the nonlocal FSM employs less degrees of freedom to obtain the accurate results. In this research, the application of BCFSM in the study of buckling and vibration behaviors of rectangular SLGSs is investigated for the first time. The vibration of SLGSs as well as their buckling behavior under uniform and non-uniform uniaxial compressive loading, uniform biaxial loading and uniform shear loading is considered. Moreover, the present methodology considers the influence of small scale effects on the vibration and bucking of SLGSs. The effects of length, aspect ratio and nonlocal parameter are considered in the results.

This paper is organized as follows. In Section 2, Eringen's nonlocal theory of elasticity and its application in classical plate theory is reviewed. In Section 3, the mathematical formulation of the bubble complex finite strip method is presented and the method is developed to study the buckling and vibration of single-layered graphene sheets. Section 4 includes the numerical results and discussions. Finally, some concluding remarks are presented in Section 5.

2. Theory

The aim of this part is to introduce Eringen's nonlocal theory of elasticity and its application in classical plate theory.

2.1. Nonlocal theory

According to the nonlocal elasticity theory [17–19], the relationship between stress and strain is expressed as

$$\sigma_{ij}(x)=\int_V \lambda(|x-x'|,\tau)C_{ijkl}\varepsilon_{kl}(x')dV(x'), \tag{1}$$

where σ_{ij}, ε_{kl} and C_{ijkl} are the stress, strain and the fourth order elasticity tensors, respectively. $\lambda(|x-x'|,\tau)$ is the nonlocal modulus or attenuation function which incorporates the nonlocal effects into the constitutive equations; $|x-x'|$ represents the Euclidean form of the distance between x and x', and τ is a material constant that depends on the characteristic length ratio l_i/l_e, where l_i is an internal characteristic length (e.g., lattice parameter, granular distance, distance between C–C bonds) and l_e is an external characteristic length (e.g., crack length, wave length). The material constant is defined as $\tau=e_0 l_i/l_e$ and could be evaluated by experimental results or molecular dynamics (MD) simulation results. Parameter e_0 strongly depends on the complicated internal microstructures of nanomaterials; it is estimated such that the relations of the nonlocal elasticity model could provide satisfactory approximations of the atomic dispersion curves of plane waves with those of the atomic lattice dynamics [52].

An equation in differential form is used as an alternative to Eq. (1), because it is difficult to deal with integral constitutive equations. This equation, which is the basis of all the nonlocal constitutive formulations, is expressed as

$$\Gamma\sigma_{ij}=C_{ijkl}\varepsilon_{kl}, \tag{2}$$

where Γ denotes the nonlocal operator and is represented by Eringen as

$$\Gamma=1-\mu\nabla^2, \tag{3}$$

in which $\mu=(e_0 l_i)^2$ and ∇^2 is the Laplacian operator. Therefore, the equation of nonlocal elasticity is represented as

$$(1-\mu\nabla^2)\sigma_{ij}=C_{ijkl}\varepsilon_{kl}. \tag{4}$$

Eq. (4) could be used in various one-, two- and three-dimensional nanoscale problems.

2.2. Nonlocal differential equation of SLGS

A SLGS could be considered as a nanoplate. Fig. 1 shows a SLGS and the corresponding rectangular continuum nanoplate model.

According to the nonlocal elasticity theory, the stress–strain relationship for a 2D isotropic nanoplate is

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}-\mu\nabla^2\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}=\frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \frac{1-\nu}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{xy}\end{Bmatrix}, \tag{5}$$

where E and ν are the modulus of elasticity and Poisson's ratio of the GS, respectively. According to the classical plate theory (CPT), the displacement field at an arbitrary point is expressed by [70]

$$\begin{aligned}u(x,y,z,t)&=u(x,y,t)-z\frac{\partial w}{\partial x}(x,y,t)\\ v(x,y,z,t)&=v(x,y,t)-z\frac{\partial w}{\partial y}(x,y,t)\\ w(x,y,z,t)&=w(x,y,t)\end{aligned} \tag{6}$$

in which u, v and w are the components of displacement vector in the mid-plane along the x, y and z directions, respectively; and t denotes the time. The strain–displacement relations are determined