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Hamiltonian system-based benchmark bending solutions of rectangular thin plates with a corner point-supported

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ABSTRACT

The benchmark bending solutions of rectangular thin plates with a corner point-supported are obtained by an up-to-date symplectic superposition method within the framework of the Hamiltonian system. The developed method offers a rational way to obtain the solutions of corner point supported thin plates with sufficient accuracy. Appropriate extension of the method can also yield more benchmark solutions of the similar problems. Comprehensive numerical results are presented for future validation of various approximate/numerical methods.

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1. Introduction

Rectangular thin plates with a corner point-supported have recently attracted the authors' attention because they are of importance in both mechanical and civil engineering. Extensive applications such as the solar panels, printed circuit boards, slate roofs, aircraft and aerospace components are frequently encountered and widely used. The lack of contributions on the benchmark bending solutions for these plates motivates the present work.

Actually, the approximate/numerical methods have been adopted to analyze the problems of some point-supported plates. Rajaiah and Rao [1] applied the collocation method to present a series solution to the problem of laterally loaded square plates simply supported at discrete points around its periphery. Shanmugam et al. [2] proposed an approximate method to predict the bending behavior of uniformly loaded rhombic and isosceles triangular orthotropic plates supported at corners based on the principle of minimizing the total potential energy and the use of a polynomial deflection function. Raamachandran and Reddy [3] developed the charge simulation method to solve the bending problem of a circular plate fixed at a number of points along its edge, which was somewhat similar to the boundary element method. Aksu and Felemban [4] used the finite difference energy method to examine the free vibration characteristics of corner point supported Mindlin plates based on the variational procedure in conjunction with the finite difference method. Azarkhin [5] found a series solution for the bending of thin plate with three-point support, with the unknown constants to be determined by the condition of minimum potential energy. A very simple closed-form expression for the deflection of the free corner was provided. Kitipornchai et al. [6] presented the solutions of free flexural vibration of corner supported Mindlin plates of arbitrary shape with a hybrid numerical approach combining the Rayleigh-Ritz method and the Lagrange multiplier method. Gutierrez and Laura [7] adopted the method of differential quadrature to analyze the transverse vibrations of rectangular thin plates with point supports. Static beam functions were introduced by Cheung and Zhou [8] and Zhou [9], respectively, to derive the eigenfrequency equations of point-supported rectangular composite plates as well as plates with variable thickness by using the Rayleigh-Ritz approach. Zhao et al. [10] studied the problem of plate vibration under complex and irregular internal support conditions via the discrete singular convolution method [11,12]. Huang et al. [13] offered a discrete green function method for free vibration analysis of rectangular plates with point supports. Altekin [14] investigated the bending of orthotropic super-elliptical plates on intermediate point supports. The Ritz method was used and the total potential energy functional was modified by introducing the Lagrange multipliers to improve the accuracy of the stress resultants.

In comparison with the prosperity of approximate/numerical solutions, analytical solutions are scarce for point-supported plates' problems. Singhal and Gorman [15] obtained the free vibration frequencies and mode shapes of partially clamped cantilevered

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rectangular plates with rigid point supports by an analytical procedure based on the method of superposition. Lim et al. [16] developed the analytical solutions for bending of rectangular thin plates supported only at its four corners by the symplectic elasticity approach [17], in which the free boundaries with corner supports were dealt with using the variational principle.

As far as the authors know, the analytical bending solutions to the title problems have not been reported in the literature, which is probably due to the mathematical complexity of such boundary value problems. Although the well-established approximate/numerical methods could cover these problems with acceptable errors, the analytical solutions provide the benchmarks, which plays an irreplaceable role in both theoretical and engineering aspects.

A novel Hamiltonian system-based symplectic superposition approach [18] is further developed in this paper to analytically provide the benchmark solutions of rectangular thin plates with a corner point-supported and its opposite edges clamped, simply supported, or one of its opposite edges clamped and the other one simply supported. The proposed approach furnishes a rational rigorous derivation to obtain the analytical solutions of the fundamental plate bending problems while avoids the numerical solution of complex transcendental equations, therefore, it combines the advantages of the symplectic approach and the superposition method. The accuracy of the present solutions are validated by those from the finite element method (FEM) via more than 200 numerical results.

2. Hamiltonian system-based governing equation

The Hellinger-Reissner variational principle for the bending problem of a thin plate is

$$\delta \Pi_{H-R} = 0 \tag{1}$$

where the generalized potential energy functional with two kinds of variables is

$$\begin{aligned} \Pi_{H-R} = \iint_{\Omega} \Big\{ & -M_x \frac{\partial^2 w}{\partial x^2} - 2M_{xy}\frac{\partial^2 w}{\partial x \partial y} - M_y \frac{\partial^2 w}{\partial y^2} \\ & - \frac{1}{2D(1-\nu^2)}\left[M_x^2 + M_y^2 - 2\nu M_x M_y + 2(1+\nu)M_{xy}^2\right] - qw \Big\} \mathrm{d}x\mathrm{d}y \\ & + \int_{C_1} M_n \left(\frac{\partial w}{\partial n} - \overline{\psi}_n\right)\mathrm{d}s - \int_{C_1+C_2} V_n (w - \overline{w})\mathrm{d}s \\ & + \int_{C_2+C_3} \overline{M}_n \frac{\partial w}{\partial n}\mathrm{d}s - \int_{C_3} \overline{V}_n w \mathrm{d}s \end{aligned} \tag{2}$$

herein q is the distributed transverse load, ν is the Poisson's ratio, D is the flexural rigidity, w is the transverse deflection of the plate midplane, M_x and M_y are the bending moments, M_{xy} is the torsional moment, Q_x and Q_y are the shear forces, V_n is the equivalent shear force. C_1 denotes the clamped edge, C_2 the simply supported edge, and C_3 the free edge; n and s are respectively the directions normal and tangential to the edge of the plate. $\overline{w}$, $\overline{\psi}_n$, $\overline{M}_n$ and $\overline{V}_n$ are the known deflection, slope, bending moment and equivalent shear force, which are all functions of the arc length of the plate edge. Assuming the independence of M_x, M_y, M_{xy} and w and the arbitrariness of their variation, Eq. (1) yields the basic equations as well as the boundary conditions of the plate.

Eq. (1) gives the natural conditions of stationary Π_{H-R}, two of which are

$$\begin{aligned} M_x &= -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \\ M_{xy} &= -D(1-\nu)\frac{\partial^2 w}{\partial x \partial y} \end{aligned} \tag{3}$$

Without regard to the line integrals, substituting Eq. (3) into Eq. (2) gives a functional denoted by Π^*:

$$\begin{aligned} \Pi^* = \iint_{\Omega} \Bigg[& \frac{D}{2}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \frac{D}{2}\left(\frac{\partial^2 w}{\partial y^2}\right)^2 + D(1-\nu)\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 \\ & + D\nu \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \frac{D}{2(1-\nu^2)}\left(\frac{M_y}{D} + \frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right)^2 - qw \Bigg]\mathrm{d}x\mathrm{d}y \end{aligned} \tag{4}$$

Define

$$\frac{\partial w}{\partial y} = \theta \tag{5}$$

and the Lagrange multiplier T, the functional Π^* is transformed into a new functional Π_H:

$$\begin{aligned} \Pi_H = \iint_{\Omega} \Bigg[& \frac{D}{2}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \frac{D}{2}\left(\frac{\partial \theta}{\partial y}\right)^2 + D\nu \frac{\partial^2 w}{\partial x^2}\frac{\partial \theta}{\partial y} + D(1-\nu)\left(\frac{\partial \theta}{\partial x}\right)^2 \\ & - \frac{D}{2(1-\nu^2)}\left(\frac{M_y}{D} + \frac{\partial \theta}{\partial y} + \nu\frac{\partial^2 w}{\partial x^2}\right)^2 + T\left(\theta - \frac{\partial w}{\partial y}\right) - qw \Bigg]\mathrm{d}x\mathrm{d}y \end{aligned} \tag{6}$$

Assuming the arbitrariness of δT, δM_y, δw and $\delta\theta$,

$$\delta \Pi_H = 0 \tag{7}$$

yields

$$\frac{\partial w}{\partial y} = \theta \tag{8a}$$

$$\frac{\partial \theta}{\partial y} = -\nu\frac{\partial^2 w}{\partial x^2} - \frac{M_y}{D} \tag{8b}$$

$$\frac{\partial T}{\partial y} = -D(1-\nu^2)\frac{\partial^4 w}{\partial x^4} + \nu\frac{\partial^2 M_y}{\partial x^2} + q \tag{8c}$$

$$\frac{\partial M_y}{\partial y} = -T + 2D(1-\nu)\frac{\partial^2 \theta}{\partial x^2} \tag{8d}$$

over the plate domain.

From Eq. (8d),

$$\begin{aligned} T &= -\frac{\partial M_y}{\partial y} + 2D(1-\nu)\frac{\partial^2 \theta}{\partial x^2} = D\left[\frac{\partial^3 w}{\partial y^3} + (2-\nu)\frac{\partial^3 w}{\partial x^2 \partial y}\right] \\ &= -\left(Q_y + \frac{\partial M_{xy}}{\partial x}\right) = -V_y \end{aligned} \tag{9}$$

which identifies the physical meaning of the Lagrange multiplier T: the opposite of the equivalent shear force V_y.

The variational principle (7) is a form of the Hamiltonian variational principle for thin plate bending.

Equations (8a–8d) are written as

$$\partial \mathbf{Z}/\partial y = \mathbf{HZ} + \mathbf{f} \tag{10}$$

where $\mathbf{H} = \begin{bmatrix} \mathbf{F} & \mathbf{G} \\ \mathbf{Q} & -\mathbf{F}^{\mathrm{T}} \end{bmatrix}$, $\mathbf{Q} = \begin{bmatrix} D(\nu^2-1)\partial^4/\partial x^4 & 0 \\ 0 & 2D(1-\nu)\partial^2/\partial x^2 \end{bmatrix}$, $\mathbf{F} = \begin{bmatrix} 0 & 1 \\ -\nu\partial^2/\partial x^2 & 0 \end{bmatrix}$, and $\mathbf{G} = \begin{bmatrix} 0 & 0 \\ 0 & -1/D \end{bmatrix}$. $\mathbf{Z} = [w, \theta, T, M_y]^{\mathrm{T}}$ is the state vector. $\mathbf{f} = [0, 0, q, 0]^{\mathrm{T}}$ is the vector with respect to the external load q. Observing $\mathbf{H}^{\mathrm{T}} = \mathbf{JHJ}$, where $\mathbf{J} = \begin{bmatrix} 0 & \mathbf{I}_2 \\ -\mathbf{I}_2 & 0 \end{bmatrix}$ is the symplectic matrix in which $\mathbf{I}_2$ is 2×2 unit matrix, $\mathbf{H}$ is a Hamiltonian operator matrix [19] thus Eq. (10) is the Hamiltonian system-based governing equation for thin plate bending.

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