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Elastic axially compressed buckling of battened columns

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ABSTRACT

This paper presents an analytical solution for the linear elastic buckling analysis of simply supported battened columns subjected to axial compressed loading. The critical buckling load is derived by using the classical energy method. Unlike most of existing work, the present approach considers not only the shear effect but also the discrete effect of battens on the global buckling behaviour of the columns. The present analytical solution is validated using the data obtained from the finite element analysis. The results show that the number of battens has significant influence on the critical buckling load of battened columns, particularly when the relative rigidity of the batten to the main member is small. It is shown that the critical buckling load increases with the number of battens, the combined bending and shear rigidity of battens, but decreases with the increased membrane stiffness of the two main members, and the increased distance between the centroids of the two main members.

1. Introduction

Built-up columns and stitched struts are widely used in steel construction especially when the effective lengths are great and the compression forces are relatively small. These columns are usually composed of two or more parallel main members interconnected by lacing or batten plates (see Fig. 1, for example). As the moment of inertia of the built-up cross section increases with the distance between the centroids of the main members, the built-up column normally has large bending rigidity and large resistance against global buckling. However, compared to the solid column with the same moment of inertia, the built-up column has weak shear stiffness and thus is more flexible, which in turn can significantly reduce its global buckling resistance.

Consideration of shear deformations in the elastic buckling analysis of columns subjected to compressive loads was first proposed by Engesser (1889), who extended Euler's buckling formula for prismatic straight columns made of anisotropic material by including shear deformations. Engesser's formula predicts an upper limit sometimes referred to as the shear buckling load as the slenderness is reduced [1]. Engesser's pioneering work was followed by Haringx (1948), who derived an alternative buckling formula which predicted an infinite buckling load as the slenderness approached zero [1]. There has been a long debate in literature on which shear deformations should be included in the analysis of column buckling (see, for example, [2–9]). Experimental data and more advanced analyses have suggested that Haringx's formula is suitable for applying to short rubber rods and helical springs, whereas Engesser's formula is appropriate for applying to sandwich columns, laced columns, castellated columns and the columns with batten plates or with perforated cover plates [1,4,5–8].

The buckling of built-up and/or battened columns has been investigated extensively in last decades. The work includes the out-of-plane buckling [10], torsional-flexural buckling [11], and interactive buckling [12] of battened columns. The buckling of battened columns with tapers [13] and the effect of shear [14,15] on the buckling behaviour of battened columns were also studied. Experimental results on the buckling of the laced and battened columns were reported in [16–18]. In addition, studies on the structural performance of built-up and battened columns under various different loadings were also accomplished [19–23].

In this paper an analytical approach using the energy method is presented to determine the critical buckling loads of battened columns. Unlike most of existing work, the present approach considers not only the shear effect but also the discrete effect of battens. A simple close-form solution for determining the critical buckling load of simply supported battened columns subjected to axial compression load is developed. The critical load derived is validated using finite element analysis methods. The present analytical solution highlights the importance of taking into account the discrete effect of battens on the global buckling of battened columns.

2. Formulation of critical buckling loads

Consider a battened column with the length $l=na$, where a is the distance between two neighbouring battens or stitches, $n+1$ is the total number of battens along the column length, as shown in

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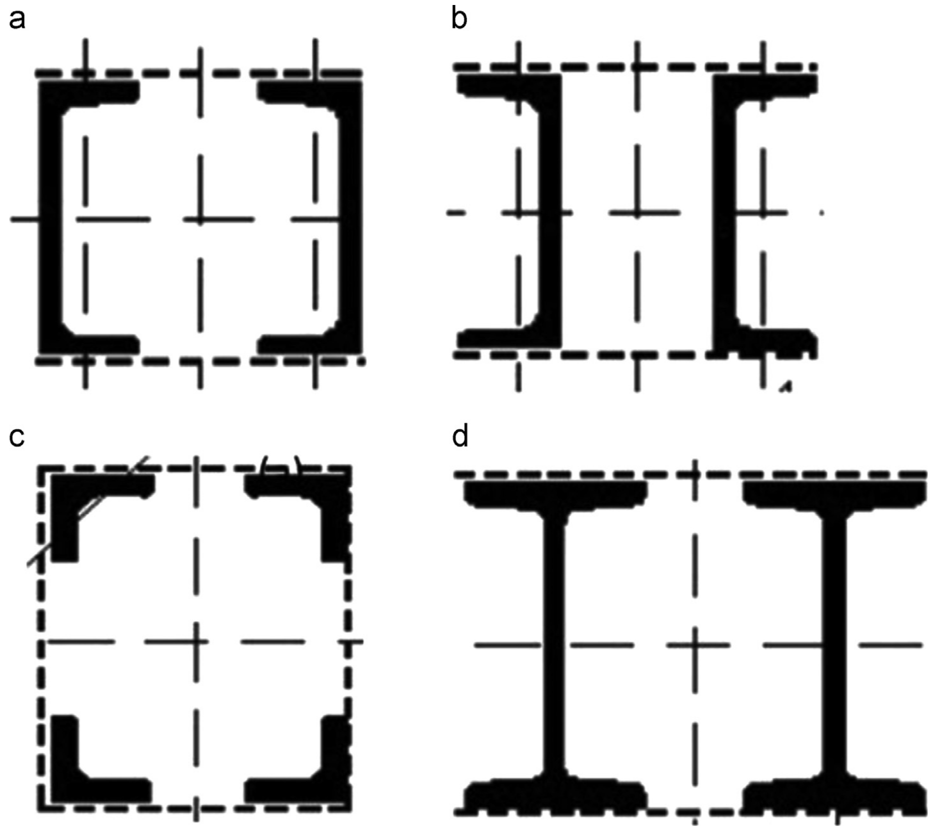

Fig. 1. Examples of built-up columns. (a, b) Channel-sections battened on flanges, (c) angle-sections battened on four sides, and (d) I-sections battened on flanges.

Fig. 2a. When it is subjected to a compressive load the column may buckle. Assume the buckling to occur only in the x–z plane for which case the buckling mode can be characterized by the longitudinal and transverse displacements of the upper and lower main members of the column. Assume that, during the buckling, the upper and lower main members deform according to Bernoulli's hypothesis. Let $u_1(x)$ and $u_2(x)$ be the axial displacements of the centroids of the upper and lower main members, and $w(x)$ be their transverse displacement (i.e. the two members have the same transverse displacement). According to the sectional displacement assumptions shown in Fig. 2b, the axial displacement at any point at a section with distance x from origin can be expressed as follows:

For the upper main member

$$u_t(x,z)=u_1(x)-(z+e)\frac{dw}{dx} \tag{1}$$

For the lower main member

$$u_b(x,z)=u_2(x)-(z-e)\frac{dw}{dx} \tag{2}$$

where e is the half-distance between the centroids of the upper and lower main members. The axial strains in the two main members can be obtained using the strain–displacement relation as follows:

For the upper main member

$$\varepsilon_t(x,z)=\frac{\partial u_t}{\partial x}=\frac{du_1}{dx}-(z+e)\frac{d^2w}{dx^2} \tag{3}$$

For the lower main member

$$\varepsilon_b(x,z)=\frac{\partial u_b}{\partial x}=\frac{du_2}{dx}-(z-e)\frac{d^2w}{dx^2} \tag{4}$$

The strain energy of the two main members due to the axial and transverse displacements can be expressed as follows:

$$U_1=\frac{E}{2}\int_o^l\int_A\varepsilon_t^2dAdx+\frac{E}{2}\int_o^l\int_A\varepsilon_b^2dAdx \tag{5}$$

where E is Young's modulus and A is the cross-sectional area of the main member. Substituting Eqs. (3) and (4) into (5), it yields

$$U_1=\frac{EA}{2}\int_o^l\left[\left(\frac{du_1}{dx}\right)^2+\left(\frac{du_2}{dx}\right)^2\right]dx+EI\int_o^l\left(\frac{d^2w}{dx^2}\right)^2dx \tag{6}$$

where I is the moment of inertia of the main member.

Let Δ be the generalized shear displacement, which is defined in terms of the strain energy of the battens caused by the axial and transverse displacements of the main members. The geometrical relation between the centroids of upper and lower main members indicates that,

$$\Delta=2e\left(\frac{u_1-u_2}{2e}-\frac{dw}{dx}\right) \tag{7}$$

It is obvious that if $\Delta=0$ the batten has rigid displacements only and therefore there is no strain energy generated. This means that the strain energy of the battens can be calculated based on the generalized shear displacement Δ.

Assume that the batten itself can be modelled as a Timoshenko beam with a length of $l_b=2e$. Owing to the rigid connections between battens and main members, the end boundaries of the batten beam have to be restrained in its rotational degree of freedom. In this case the relative deflection between the two ends of a batten beam when subjected to a pair of unit loads in opposite directions at its ends can be expressed as follows:

$$\delta_b=\frac{l_b}{GA_s}+\frac{l_b^3}{12EI_b} \tag{8}$$