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Adaptable dies to minimize distortion in non-axisymmetric extrusions

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ABSTRACT

An upper bound model for the extrusion of a round billet into a round-corner rectangular product has been developed for use with the adaptable die design method. The analysis shows the use of the adaptable die design method for minimizing distortion in the extrusion product. The flow fields are a modification of previous axisymmetric models, but are extended to permit rotational movement to occur during deformation. The extrusion die is analyzed in two segments. A streamlined segment imposes the deformation, and a bearing length segment is attached at the end. The power terms associated with the deformation process are derived. Similar to the axisymmetric analysis, the use of a streamlined die shape at the entrance and exit regions for the deformation zone produces no internal shear power losses along the two internal surfaces, which separate the initial and final rigid body material from the deformation region. The extrusion die shapes are determined by the adaptable die design method, where adjustable coefficients for characterizing the streamlined die are found by minimizing the volumetric effective strain rate deviation. The adaptable extrusion die shapes, based on minimizing the volumetric effective strain rate deviation in the deformation zone, produce minimum distortion in the extrusion product and are equivalent to a controlled strain rate die shape. Confirmation is obtained by analyzing the results from three-dimensional finite element models for extrusion through the adaptable die shapes. The analysis presented can also be modified to analyze other non-reentrant extrusion product shapes.

1. Introduction

Warping, twisting, underfill, or distortion of the final product during the extrusion of non-axisymmetric shapes remains a challenging problem within the metal forming industry. Die designers are continuously seeking new methodologies to select the die shape to reduce distortion in the extrudate (extruded product). In previous papers, the authors have used the adaptable die design method to analyze axisymmetric extrusion processes [1–3] and determine die shapes that minimize distortion during axisymmetric extrusion [4,5]. In the present paper, the analysis method is extended to a non-axisymmetric (termed three-dimensional) extrusion. The process that is examined herein is the extrusion of a round-cornered rectangular shape from a cylindrical billet. The adaptable die design method allows the determination of the extrusion die shape that produces a product with minimal distortion.

The same extrusion process parameters and finished product properties in axisymmetric extrusion are considered important in extrusion of non-axisymmetric shapes. However, the effect of each variable is complicated by a possible rotational flow component in the deformation zone. As a result, the distortion and strain distribution in the finished product, as well as the strain rate distribution in the deformation zone, can vary to a much greater degree than in the axisymmetric flow. Many of the experimental and analytical techniques used extensively in axisymmetric extrusion have been applied to extrusion of non-axisymmetric shapes. However, the relationship between the die shape and the three-dimensional extrusion process and product parameters remains limited.

1.1. Background

Both the upper bound and finite element techniques have been used to analyze three-dimensional extrusion. An early paper by Nagpal [6] proposed the use of dual-stream functions to establish a velocity field that satisfied the incompressibility and boundary condition requirements for an upper bound analysis. His approach provided one of the first upper bound analyses for non-axisymmetric extrusion. The body of upper bound and finite element work grew through the efforts of Basily and Sansome [7], Boer et al. [8], and Boer and Webster [9]. They analyzed a simple drawing where a rod is pulled through a die, going from round to square through straight converging dies.

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Nomenclature

A_f	final cross-sectional area of product
A_i	coefficient for the Legendre polynomials in the ε function
A_r	half-length of major side of rectangular product
B_i	coefficient for the series expansion used in the γ function
B_r	half-length of minor side of rectangular product
b, c	parameters in Eq. (4)
C_i	coefficient for the Legendre polynomial representation of the streamlined die segment
E, H	parameters in strain rate components — Eqs. (38)–(43)
i, k	indices
J^*	externally supplied power
K_c, K_{c2}, K_{c3}	parameters in Eq. (13)
L	length of streamlined die segment
L_B	length of bearing die segment
M_k	coefficient for the Legendre polynomial representation of higher-order C_i
m_f	constant friction factor ($0.0 \le m_f \le 1.0$)
n_a	order of the ε function
n_b	order of the γ function
n_c	order of the ψ function (die surface) in the streamlined die segment
n_m	order of the C_i Legendre polynomial expansion
P_i	Legendre polynomial
p_{avg}	average extrusion pressure
Q	parameter in Eq. (55)
R_{f-eq}	equivalent radius of final product
R_o	radius of original billet
R_r	corner radius of rectangular product
r	radial coordinate
r_B	radial position of streamlined die segment to bearing segment transition point
r_f	radial position of the Γ_2 surface
r_o	radial position of the Γ_1 surface
S	linear distance of point on product surface from centerline axis
S_m	maximum value for S (*i.e.* maximum distance from point on product surface to centerline axis)
$\dot{U}_r$	radial component of velocity for a point in the deformation region
$\dot{U}_\theta$	angular component of velocity for a point in the deformation region
$\dot{U}_\phi$	rotational component of velocity for a point in the deformation region
v_o	velocity of original billet
v_f	velocity of final product
v_r	base radial velocity in deformation region
$\dot{W}_i$	internal power of deformation in the deformation region
$\dot{W}_{S\Gamma_1}$	shear power losses along the Γ_1 surface
$\dot{W}_{S\Gamma_2}$	shear power losses along the Γ_2 surface
$\dot{W}_f$	friction power losses along the die-workpiece interface
x, y	independent variables in Legendre polynomial
α	angular position of entrance point of the streamlined die segment
α_B	angular position of streamlined die segment to bearing segment transition point
α_{Br}	angular position of mid point of the major side of the rectangular product at the die exit
α_f	angular position of any point on the rectangular product at the die exit
Γ_1	first surface of velocity discontinuity, separating the incoming billet from the deformation zone
Γ_2	second surface of velocity discontinuity, separating the deformation zone from the final product
γ	function to allow radial velocity flexibility in the θ direction
Δv_1	tangential velocity difference along the Γ_1 surface
Δv_2	tangential velocity difference along the Γ_2 surface
ε	function to allow radial velocity flexibility in the r direction
$\dot{\varepsilon}_{rr}$	normal strain rate component in the radial direction
$\dot{\varepsilon}_{\theta\theta}$	normal strain rate component in the angular direction
$\dot{\varepsilon}_{\phi\phi}$	normal strain rate component in the rotational direction
$\dot{\varepsilon}_{r\theta}$	shear strain rate component
$\dot{\varepsilon}_{\theta\phi}$	shear strain rate component
$\dot{\varepsilon}_{\phi r}$	shear strain rate component
θ	angular coordinate
μ	side length ratio of rectangular product
Π_A, Π_B, Π_C	parameters in strain rate components — Eqs. (38)–(43)
ρ	relative radial coordinate (r/r_o).
ρ_B	relative radial position of streamlined die segment to bearing segment transition point
σ_o	flow stress of the workpiece
Φ_A, Φ_B	parameters in strain rate components — Eqs. (38)–(43)
ϕ	rotational coordinate
ϕ_1, ϕ_2	rotational positions for start and end of the corner rounding for the rectangular product
ϕ_m	rotational position for the point of the product surface that is the maximum distance from the centerline axis
ψ	angular position of the die as a function of radial position
Ω	function to allow rotational flow in the deformation region

Kiuchi [10] studied non-axisymmetric extrusions, including re-entrant geometries, through straight converging dies. Re-entrant extrusions are shapes in which a line drawn radially outward from the centroid of the finished cross section intersects the outer shape of the cross section at more than one point. Gunasekera and Hoshino [11–13] developed an upper bound model to study the drawing or extrusion of polygonal shapes (shapes with n-equal sidelengths) through straight converging as well as through streamlined dies. Their work illustrated that the original plane of a split billet becomes distorted upon extruding to a non-axisymmetric shape. Wu and Hsu [14] developed a flexible velocity field to extrude polygonal shapes through straight converging dies. Collectively, most of these papers [6–14] focused on the optimum die length to minimize the total drawing or extrusion force, but lacked flexibility in the velocity fields to account for friction (except for the work by Wu and Hsu [14]).

The next set of papers developed upper bound and finite element approaches to extrude any non-axisymmetric shape with flexibility to account for friction. Han et al. [15] created a velocity field from their previous axisymmetric upper bound model [16] in order to study extrusion through streamlined dies that produced clover-shaped sections. Yang et al. [17] further applied this general upper bound model to study extrusion of elliptic and rectangular sections. Fig. 1 shows the resulting distorted grid on the square and rectangle symmetry planes from their upper bound model.