International Journal of Mechanical Sciences 77 (2013) 227–238

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Free vibration analysis of Lévy-type functionally graded spherical shell panel using a new exact closed-form solution

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ARTICLE INFO

Article history:
Received 18 June 2011
Received in revised form
29 July 2013
Accepted 7 October 2013
Available online 17 October 2013

Keywords:
Exact closed-form frequency equation
Spherical shell panel
First-order shear deformation theory
Donnell and Sanders shell theories

ABSTRACT

An exact closed-form analysis for describing the natural vibrations of a FG moderately thick spherical shell panel is developed. The strain–displacement relations of Donnell and Sanders theories are used to obtain the exact solutions. The shell has two opposite edges simply supported (i.e., Lévy-type). The material properties change continuously through the thickness of the shell, which can vary according to a power-law distribution of the volume fraction of the constituents. The new auxiliary and potential functions are employed to exactly decouple the governing equations of the vibrated spherical shell panel, leading to the exact closed-form frequency equation in the form of determinant. The accuracy and validity of the solutions are established with the aid of a 3D finite element analysis as well as by comparing the results with the data reported in the literature. The effects of various stretching–bending couplings on the frequency parameters are discussed.

1. Introduction

Because of the curvature of the middle surface, spherical shell panels are very stiff for both in-plane and bending loads. Therefore, they are largely used in many engineering structures, including pressure vessels, ship hulls, containers of liquids, pipes, missiles and many other structures. It is noticeable that the analysis of shells has difficulty related to the curvature, which is also the reason for the carrying load capacity of these structures. These shells are often subjected to dynamic loads that cause vibrations. Hence, it is very important to have an accurate procedure for the free vibration analysis of such structures.

Many of the shell theories were developed based on the classical shell theory (i.e., CST) and the linear elasticity concepts. Love was the first investigator to present a successful approximation thin shell theory based on the classical linear elasticity [1]. The Love theory of thin elastic shells is also referred to as the first-order approximation shell theory. Donnell [2] established the nonlinear theory of circular cylindrical shells under the simplifying shallow-shell hypothesis. Because of its relative simplicity and practical accuracy, this theory (referred to as Donnell's nonlinear shallow-shell theory) has been widely used. But, the equations of this theory are obtained by neglecting the in-plane inertia, transverse shear deformation and rotary inertia, giving accurate results only for very thin shells. Also, Donnell [3], Vlasov [4], and Mushtari [5] independently developed a simplified engineering theory of thin shells of a general form (this is also referred to as Donnell–Mushtari–Vlasov theory of thin shells). Sanders [6] developed the first-order-approximation shell theory from the principle of virtual work and by applying the Kirchhoff–Love assumptions. Sanders theory of thin shells has removed successfully the inconsistencies of the Love theory. Naghdi [7] analyzed the accuracy of the Love–Kirchhoff theory of thin elastic shells. The interested reader who wants to be more acquainted with the shell's history in more details is referred to other works [8,9].

Functionally graded materials (FGMs) are special composites with material properties that vary continuously through their thickness. Typically, FGMs are made of a ceramic and a metal in such a way that the ceramic can resist the severe thermal loading from the high temperature environment, whereas the metal is served in order to decrease the large tensile stress occurring on the ceramic surface at the earlier stage of cooling. Also, the gradual change of the material properties avoids discontinuities of stresses.

According to the aforementioned FG properties, spherical shell panels made of FGMs are of great interest for engineering design and manufacture, recently. But, a beneficial literature review reveals that analytical and numerical studies on FG spherical shell panels are, however, rare whereas several studies have been performed to analyze the mechanical, thermal or the thermo-mechanical responses of isotropic spherical shell panels. Thus, it is important to understand the exact dynamic behavior of functionally graded (FG) spherical shell panels.

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0020-7403/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijmecsci.2013.10.008

In many applications of FG spherical shell panels, the thickness of the shell in comparison with its side is significant and the thickness to side ratio takes from 0.05 to 0.2. These shells are known as the moderately thick FG spherical shell panels. Also, it has long been known that the classical Love–Kirchhoff shell theories are valid only for thin shells and give proper results for lower frequencies. In addition, these theories underestimate deflections and overestimate frequencies. In order to have a reliable prediction of the response of moderately thick FG spherical shells and to eliminate the deficiency of the CST, the first-order shear deformation theory (FSDT), including the effects of transverse shear deformation and rotary inertia, should be employed.

When the equations of motion of a moderately thick FG spherical shell panel are derived by using the FSDT of shells, these partial differential equations must be solved through a type of solution method namely numerical methods (e.g., finite element method, differential quadrature method, Galerkin method and so on), semi-analytical methods or exact analytical methods. However, owing to the mathematical and computational complexities of exact solutions, a wide range of research has been carried out on free vibration of spherical shell panels which mostly used a numerical solution method. Tornabene and Viola [10] studied the dynamical behavior of spherical shell panels using the FSDT. The numerical solutions have been computed by means of the technique known as the Generalized Differential Quadrature (GDQ) Method. Xiang et al. [11] analyzed the free vibration of laminated composite spherical shells through the FSDT and a meshless global collocation method. Ferreira et al. [12] applied the FSDT of Donnell to evaluate the natural frequencies of cross-ply composite spherical shells through a meshless method. Zenkour [13] investigated the static and dynamic responses of anisotropic spherical shells under a uniformly distributed transverse load using a refined mixed first-order shear deformation theory and a numerical method. Based on a finite element approach, the axisymmetric and non-symmetric vibrations of spherical shells are analyzed using the thick shell theory by Gautham and Ganesan [14]. Lim et al. [15] employed the Ritz method to analyze the free vibration of spherical shell with variable thickness using 2D shell theory. Pradyumna and Bandyopadhyay [16] analyzed free vibration of isotropic and functionally graded spherical shell panels using a higher-order formulation. A C° finite element formulation is used to carry out the analysis. Matsunaga [17] analyzed natural frequencies and buckling stresses of cross-ply laminated composite shallow shells using a two-dimensional higher-order theory and the method of power series expansion of displacement components. The elasticity solutions for free vibration analysis of spherical shell panels of rectangular planform are carried out by Liew et al. [18]. The p-Ritz method is employed to solve the problem. Also, using a numerical method, the 3D analysis is presented for free vibration of spherical shell segments with variable thickness by Kang and Leissa [19]. Redekop [20] developed the differential quadrature method to determine the natural frequencies of vibration of thick orthotropic spherical shells consisting of a material having a radial variation of properties using the linear three-dimensional theory of elasticity. Fan and Luah [21] presented the free vibration analysis of arbitrary thin shell structures using a newly developed spline finite element. Khare et al. [22] developed a simple C° isoparametric finite element formulation based on a shear deformable model of higher-order theory using a higher-order facet shell element for the free vibration analysis of isotropic, orthotropic and layered anisotropic composite and sandwich laminates. Chern and Chao [23] conducted a three-dimensional vibration analysis for a variety of simply supported shallow spherical, cylindrical, plate, and saddle (hyperbolic) panels in rectangular planform. An energy variational approach according to minimum total potential energy is used to analyze the problem.

Obviously, all research groups would like to present exact closed-form solutions for their problems. However, the presentation of this solution for the vibration problem of isotropic and FG spherical shell panels was limited to shells with simply supported boundary conditions (the Navier-type solution) [24–28]. Chaudhuri and Kabir [24] presented a Navier-type solution for free vibration of a general cross-ply doubly curved panel of rectangular planform using the four classical shallow shell theories. The paper presented by Rath and Das [25] is one of the first investigations on the vibration of shells using higher order shear deformation of theories. They presented the shell equations in curvilinear orthogonal coordinates for certain types of layered shells, the effects of shear deformations being included. The natural frequencies are calculated for symmetrically layered and non-symmetric cross-ply cylindrical shells having freely supported ends. Based on the FSDT and using the Sanders shell theory, Reddy [26] presented an exact solution for vibration and buckling problems of a simply supported cross-ply laminated spherical shell panel, under sinusoidal, uniformly distributed, and concentrated point load at the center. Also, Reddy and Liu [27] developed a higher-order shear deformation theory for shells laminated of orthotropic layers. The theory is a modification of the Sanders theory. They presented the Navier-type exact solutions for bending and natural vibration of spherical shells. Lee and Reddy [28] developed the third-order shear deformation theories of laminated composite shells using the strain–displacement relations of Donnell and Sanders theories. They presented an analytical (Navier) solution for vibration suppression in cross-ply laminated composite shells with surface mounted smart material layers using the linear versions of the two shell theories and for simply supported boundary conditions. It is noticeable that the formulation by Reddy and Liu [27] is inaccurate for deeper and thicker shells. This fact is confirmed by Carrera [29] and Qatu [30]. Biglari and Jafari [31] used a refined general-purpose sandwich panel theory for the free vibration analysis of simply supported spherical sandwich shells. Cinefra et al. [32] presented a closed form solution for free vibration of simply supported multilayered shells made of functionally graded materials using a variable kinematic shell model. Pagani et al. [33] obtained an exact dynamic stiffness formulation using one-dimensional higher order theories for free vibration analysis of thin-walled structures. Qatu [34] presented accurate natural frequencies of simply supported shallow shells for various shapes including spherical, cylindrical and hyperbolic paraboloidal shells.

To the best of the authors' knowledge, there is no literature for exact closed-form solutions of vibration analysis of Lévy-type FG spherical shell panels based on the FSDT and the authors attempt to fill this apparent void.

The main objective of this paper is to obtain the exact closed form solution for free vibration of moderately thick FG spherical shell panels based on Sanders and Donnell shell theories. For this purpose, a new exact analytical approach is developed to exactly solve the governing equations of the shell without any usage of approximate methods. To demonstrate the superiority and accuracy of obtained results, the natural frequencies are compared with the available data in literature and a finite element method (FEM) analysis. By comparing the obtained results based on Sanders and Donnell theories with a 3D elasticity analysis [23] and the 3D finite element model, it is found that Sanders theory can accurately and efficiently predict the vibration modes of FG spherical shell panel. Various coupling effects on the frequency parameters are carried out for different boundary conditions and panel geometry parameters using Sanders and Donnell shell theories.

2. Analysis

2.1. Geometrical configuration

Consider a moderately thick FG spherical shell panel of length a, width b, uniform thickness h and mean radius R. The shell has