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Mechanical properties of lattice materials via asymptotic homogenization and comparison with alternative homogenization methods

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ABSTRACT

Several homogenization schemes exist in literature to characterize the mechanics of cellular materials. Each one has its own assumptions, advantages, and limitations that control the level of accuracy a method can provide. There is often the need in heavy multiscale analyses of lattice materials to find the method that can provide the best trade-off between accuracy and computational cost.

In this paper, asymptotic homogenization (AH) is used as a benchmark to test the accuracy of alternative schemes of homogenization applied to lattice materials. AH is first applied to determine the effective elastic moduli and yield strength of six lattice topologies for the whole range of relative density. Yield surfaces are also obtained under multiaxial loading for square, hexagonal, and Kagome lattices, and closed-form expressions of the yield loci are provided for a convenient use in multiscale material problems. With respect to the relative density, the results are then compared to those obtained with other methods available in literature. The analysis shows that the latter can predict the elastic constants with an error below 10% for $\rho < 0.25$, whereas for the yield strength the discrepancy is above 20% for $\rho \geq 0.1$ due to the model assumptions. The results of this work on the effective properties of lattice materials provide not only handy expressions of prompt use in multiscale design problems, but also insight into the level of accuracy that alternative homogenization techniques can attain.

1. Introduction

Cellular solids are widely used in applications where weight savings and multifunctional properties are critical. Aerospace sandwich panels, vibration and sound insulators, compact heat exchangers, and biomedical implants are only a few examples of applications. A periodic cellular material can be considered as a microtruss obtained by tessellating a unit cell along independent periodic vectors. Periodic cellular materials with high nodal connectivity have a much higher stiffness and strength per unit mass compared to their counterparts, which have low nodal connectivity with cells arranged in either a regular or disordered pattern, such as in foams [1,2].

In a cellular material, the characteristic length of the unit cell is generally considered several orders of magnitude below the characteristic length of the component. Yet, the study of the unit cell is essential to understand the mechanical response of the material at the macroscale. Several analytical and numerical approaches, as well as experimental investigations, have been proposed in the literature to determine the mechanical properties of cellular materials [3–24]. Their main goal is to obtain the properties of the macromaterial in terms of the effective properties of its unit cell, which is a process of homogenization set to circumvent a detailed – often impractical – analysis of the entire cellular microstructure. Noteworthy contributions in the area of cellular materials are those of Gibson and Ashby [3], Masters and Evans [4], Christensen [5], and Wang and McDowell [6,25], which provide closed-form expressions of the effective mechanical properties. These methods rely on certain premises. They generally assume the cell walls behave like Euler–Bernoulli beams, examine the individual cell wall and determine the elastic constants of the cell by solving deformation and equilibrium problems. They work well for topologies that have a simple arrangement of the cell members, but present limitations if the geometry of the unit cell has a complex topology.

More recently, matrix-based techniques using the Bloch's theorem and the Cauchy–Born hypothesis have been used to homogenize the properties of planar lattice materials [26,27]. Hutchinson and Fleck [26] first formulated the microscopic nodal deformations of a lattice in terms of the macroscopic strain field,

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from which the material macroscopic stiffness properties are derived. A methodology was proposed to characterize cell topologies with a certain level of symmetry, e.g. the Kagome lattice and the Triangular–Triangular lattice. Elsayed and Pasini [27] and Elsayed [28] extended this method to deal with planar topologies that can possess any arbitrary geometry of the cell. Vigliotti and Pasini [29-31] presented a more general matrix-based procedure for the analysis of arbitrary bidimensional and tridimensional cell topologies with open and closed cells.

Other models have been proposed to model the cellular microstructure as an equivalent micropolar medium [8–13]. In micropolar elasticity, in addition to the translational deformation, an independent microscopic rotational field is usually introduced [32,33]. For a given cell topology, the micropolar elastic constants of the stiffness matrix are obtained through either an explicit structural analysis of the representative unit cell [11,12] or an energy approach [7–10].

Discrete homogenization techniques have been also developed and successfully applied to the characterization of cellular materials [34–38]. With these approaches, the lattice cell walls are modeled with discrete elements such as beam or rod elements. Taylor's expansion of nodal displacement and internal forces are derived and inserted into the equilibrium equations. The discrete sum of equilibrium equations is finally converted into a continuous relation of stress and strain from which the homogenized properties are obtained.

Among numerical approaches, asymptotic homogenization (AH) theory has been successfully applied to predict the effective mechanical properties of materials with a periodic microstructure [14–16,39]. AH has been widely used not only for the analysis of composite materials and topology optimization of structures [40–43], but also for the characterization of porous materials, such as tissue scaffolds [17,44–46]. AH assumes that any field quantity, such as the displacement, can be described as an asymptotic expansion, which replaced in the governing equations of equilibrium allows to derive the effective properties of the material [14,16]. Validation of AH results with experiments has shown that AH is a reliable and accurate method to predict the effective mechanical properties of heterogeneous periodic materials [47–52]. Takano et al. [47] applied AH to analyze micro-macro coupled behavior of the knitted fabric composite materials under large deformation conditions. The predicted largely deformed microstructures were compared with the experimental results, and a very good agreement was observed. In another study, to validate the accuracy of AH results, the predicted value of the effective elastic modulus of a porous alumina with 3.1% porosity was compared with the elastic constant measured from experiments [48], and a relative error of 1% was found. Guinovart-Díaz et al. [49,50] computed the thermoelastic effective coefficients of a two-phase fibrous composite using AH. The results were compared with experimental data, and a good agreement was found. In a more recent study, AH has been applied to predict the failure behavior of three-dimensional weaved composites [52]. The stress–strain response and failure modes of the composite was modeled and shown to match with experimental results. Compared to other homogenization schemes, a noteworthy advantage of AH is that the stress distribution in the unit cell can be determined accurately and thus be used for a detailed analysis of the strength and damage of the heterogeneous periodic materials [38,53,54]. Furthermore, AH has neither limitation on the cell topology nor on the range of relative density; essentially, AH can handle any lattice regardless of its relative density. Here for the first time, AH is applied for the characterization of lattice structures. The results are then considered as a benchmark for the comparison with other homogenization methods.

As briefly described above, several methods exist in literature to model the mechanics of cellular materials. Each one has its own assumptions, advantages, and limitations. One central issue to the process of solving a given problem is the careful selection of the homogenization technique that is most effective in terms of accuracy and the most convenient as far the computational cost is concerned. This task, however, presents often trade-offs, which are not always properly defined. For example, closed-form expressions, such as those in [3,6,28,29], can be conveniently used to fast compute the effective properties of a lattice material; however, problems of accuracy might emerge if the microstructure does not respect the model assumptions. The hypothesis that cell walls behave like beam/rod elements ceases to provide reliable results for increasing values of relative density. Moreover, the Euler–Bernoulli beams and rod elements cannot capture the deformation of the solid material at the cell joints, a problem that might affect the estimation of the yield strength of the material. Other techniques, on the other hand, have been proved to be accurate over the whole range of relative densities, but for certain problems they might have the drawback to require longer time of computation [54]. This represents a curb in large multiscale optimization problems of lattice materials [55,56], because the material properties must be iteratively evaluated several times. In this case, the trade-off solution would be to prefer a method which is computationally faster, as long as it is used in a range of relative density where the results are considered satisfactory within an acceptable range of error. It is thus essential to be able to contrast the validity of alternative homogenization schemes so as to select the most effective method to solve a given problem.

The main goal of this work is to provide a comparative study on the accuracy of classical and more recent homogenization techniques [3,6,28,29] with respect to the governing variable of the lattice, the relative density. Due to its proved superiority, we chose AH as a benchmark method and review its fundamentals in the next section. Six lattice topologies (Fig. 1), representative of either bending or stretching dominated behavior, are selected to test the methods in characterizing their effective elastic constants and mechanical strength. For three common cell topologies, square, hexagonal, and Kagome, AH is used to obtain yield surfaces under multiaxial loading condition. As part of the analysis, approximated closed-form expressions of the results are also given in Section 3 for a practical use in multiscale analysis and design problems of cellular materials [57–59]; in particular, they might serve for (i) rapid calculation of the effective mechanical properties of lattice materials, (ii) validation purposes of experimental data [21,50],

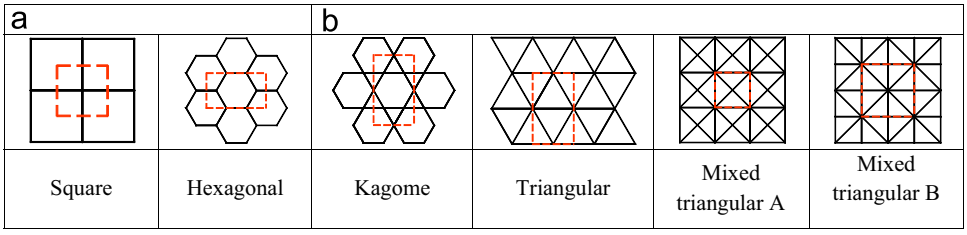

Fig. 1. Cell topologies with the associated RVE considered in this study. (a) Bending dominated (b) Stretching dominated.