International Journal of Mechanical Sciences 77 (2013) 263–268

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

The analytical solution of mixed convection heat transfer and fluid flow of a MHD viscoelastic fluid over a permeable stretching surface

M. Turkyilmazoglu *

Department of Mathematics, Hacettepe University, Beytepe, 06532 Ankara, Turkey

ARTICLE INFO

Article history:
Received 27 March 2012
Received in revised form
10 October 2013
Accepted 12 October 2013
Available online 18 October 2013

Keywords:
Non-Newtonian fluids
Mixed convection
Heat transfer
Porous stretching surface
Magnetohydrodynamics
Dufour and Soret effects

ABSTRACT

In this paper we investigate structure of the solutions for the MHD flow and heat transfer of an electrically conducting, viscoelastic fluid past a stretching vertical surface in a porous medium, by taking into account the diffusion thermo (Dufour) and thermal-diffusion (Soret) effects. It is shown that the porosity, magnetic, convection and concentration buoyancy effects can be combined within a new parameter called here as a porous magneto-convection concentration parameter. Heat transfer and concentration analysis are also carried out for a boundary process. The physical parameters influencing the flow field are viscoelasticity, porous magneto-convection concentration and suction/injection, and those affecting the temperature field are Prandtl and Dufour numbers, and further affecting the concentration field are Prandtl, Lewis and Dufour numbers. Such parameters greatly alter the behavior of solutions from unique to multiple and determine the boundaries of existence or nonexistence of solutions. The features of the skin friction coefficient, Nusselt number and Sherwood number are also easy to gain from the derived equations.

1. Introduction

Transport processes have been the main motivation in the recent research for the transport of heat through a porous medium. A great deal of fields feeds this interest in the transport through porous media, see for example Hayat et al. [1] for the updated industrial applications. The effects due to Soret and Dufour were detected to considerably influence the flow problem at hand, see Refs. [2,3] and also refer to the recent books of Bejan et al. [4], Ingham and Pop [5], Nield and Bejan [6] and Vadasz [7].

We mention here several latest publications on MHD mixed convection heat transfer and fluid flow over a stretching vertical surface in a porous medium filled with a Non-Newtonian fluid, among them are by Sahoo and Do [8] and Kumaran and Tamizharasi [9]. The works by Turkyilmazoglu [10–12] investigate the multiplicity of the solutions over both stretching and shrinking surfaces. Some more features are also presented by Turkyilmazoglu [13,14].

All of the above investigations, except perhaps [1,2], are, however, confined to forced convection flow of viscoelastic fluids, i.e. without considering the buoyancy and concentration effects. The purpose of this paper is, therefore, unlike the approximate analytical or numerical works of [1,2], to analytically investigate the steady MHD mixed convection over a stretched sheet when Soret and Dufour's effects are active in the fluid. It was found that for the existence of exact analytical solutions, Soret and Dufour's effects are governed by a relation connected to the Prandtl number. The flow, temperature and concentration fields together with the skin friction coefficient, Nusselt number and Sherwood number are easy to understand from the extracted exact analytical formulae. Multiple solutions are presented in closed-form formulae for the second grade and Walters' liquid B fluids of viscoelastic type.

The arrangement of the paper is done in the subsequent manner. The problem is stated in Section 2. Section 3 contains the closed-form solution of flow, temperature and concentration fields. Section 4 mentions the findings and discussions. Conclusions are finally outlined in Section 5.

2. Formulation of the problem

We consider the steady two-dimensional boundary layer flow due to the stretching of a heated or cooled vertical surface, see Fig. 1 for the configuration, of variable temperature $T_w(x)$ and concentration $C_w(x)$ in a permeable and electrically conducting viscoelastic fluid. A uniform magnetic field of strength B_0 is further imposed along the y-axis, which produces a magnetic effect in the x-direction. The sheet is assumed to be stretched with the velocity $u_w(x)$. Omitting heat generation and viscous dissipation, the governing equations are given by, see Hayat et al. [1], Tsai and

* Tel.: +90 3122977850; fax: +90 3122972026.
E-mail address: turkyilm@hotmail.com

0020-7403/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.ijmecsci.2013.10.011

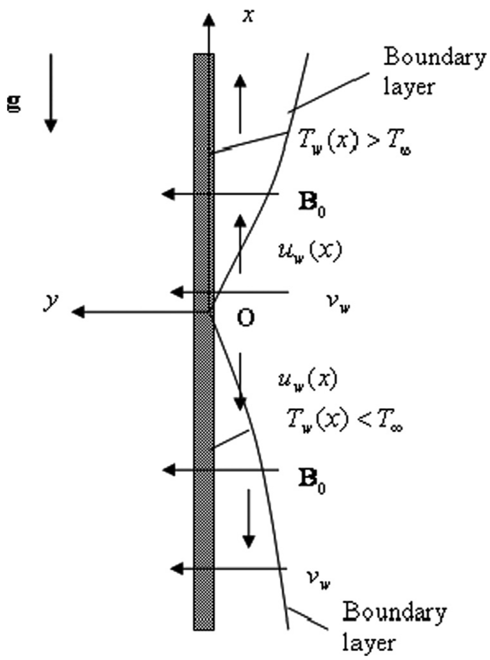

Fig. 1. The geometry of physical flow.

Huang [2] and Mushtaq et al. [3],

$$u_x + v_y = 0,$$
$$uu_x + vu_y = \nu u_{yy} - k_0((uu_{yy})_x + u_y v_{yy} + vu_{yyy}) - \sigma B_0^2 u - \frac{\nu}{\gamma_1}u + g[\beta_T(T-T_\infty) + \beta_C(C-C_\infty)], \tag{2.1}$$

$$uT_x + vT_y = \alpha T_{yy} + \frac{Dek_T}{C_s C_p}C_{yy},$$
$$uC_x + vC_y = DeT_{yy} + \frac{Dek_T}{T_m}T_{yy} \tag{2.2}$$

supplemented with the boundary conditions

$$u = u_w(x) = ax, \quad v = v_w, \quad T = T_w(x) = T_\infty + bx,$$
$$C = C_w(x) = C_\infty + cx \text{ at } y = 0,$$
$$u \to 0, \ u_y \to 0, \quad T \to T_\infty, \quad C \to C_\infty \text{ as } y \to \infty, \tag{2.3}$$

u and v are the velocity components, v_w is the mass flux velocity with $v_w < 0$ for suction and $v_w > 0$ for injection respectively. Moreover, the fluid temperature is T, the concentration is C, the acceleration due to gravity is g, the thermal diffusivity is α, the coefficient of thermal expansion is β_T, the coefficient of concentration is β_C, the modulus of the viscoelastic fluid is k_0. Moreover, ν is the kinematic viscosity and σ is the electrical conductivity. The induced magnetic field is neglected. Further $a(>0)$, b and c are constants.

3. Solution of the flow field

Introducing the similarity transformations and dimensionless variables η, $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$

$$\eta = y\sqrt{\frac{a}{\nu}}, \quad u = axf'(\eta), \quad v = -\sqrt{a\nu}f(\eta), \quad \theta = \frac{T-T_\infty}{T_w - T_\infty},$$
$$C = \frac{C-C_\infty}{C_w - C_\infty}, \tag{3.4}$$

greatly facilitates the subsequent analysis. As a result, the wall mass transfer velocity becomes $u_w = -\sqrt{a\nu}s$, and the full equations reduce into the following form:

$$f''' + ff'' - f'^2 + k(ff^{(4)} - 2f'f''' + f''^2) - Mf' - \gamma f' + \Lambda(\theta + N\phi) = 0,$$
$$pt\theta'' + Pr(f\theta' - f'\theta + D_f\phi'') = 0,$$
$$\theta'' + Le[Pr(f\phi' - f'\phi) + Sr\theta''] = 0, \tag{3.5}$$

accompanied by the reduced boundary conditions

$$f(0) = s, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1$$
$$f'(\infty) = 0, \quad f''(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0, \tag{3.6}$$

here $\Pr = \nu/\alpha$ and $Le = \alpha/D_e$ are respectively the Prandtl and Lewis numbers, $k = k_0 a/\nu$ is the viscoelastic parameter, s is the wall mass transfer parameter showing the strength of the mass transfer at the sheet $M = \sigma B^2/a\rho$ is the magnetic interaction parameter. The concentration buoyancy parameter N, the porosity parameter $\gamma = \nu/a\gamma_1$, the Dufour number D_f and the Soret number Sr are respectively defined as

$$N = \frac{\beta_C}{\beta_T}\frac{C_w - C_\infty}{T_w - T_\infty}, \quad D_f = \frac{Dek_T}{C_s C_p \nu}\frac{C_w - C_\infty}{T_w - T_\infty}, \quad Sr = \frac{Dek_T}{T_m \alpha}\frac{T_w - T_\infty}{C_w - C_\infty}.$$

Additionally, $\Lambda = dGr_x/Re_x^2$ is the constant mixed convection parameter with $Gr_x = (g\beta_T(T_w - T_\infty)x^3)/\nu^2$ being the local Grashof number and $Re_x = u_w(x)x/\nu$ is the local Reynolds number. It should be noticed that owing to the presumed form of wall conditions for the variables, as seen from Eq. (2.3), it is clear that N, D_f and S_T are no longer functions of x. Moreover, Λ is not a function of x either, after the definition of local Grashof and Reynolds numbers in the paper, which is simplified as $\Lambda = dg\beta_T$, where $d = b/a$.

We remind that $\Lambda = 0$ shows forced convection flow, whereas $\Lambda > 0$ means assisting flow and $\Lambda < 0$ indicates cooled plate respectively. It is also noted that $k < 0$ is for a second grade fluid and $k > 0$ indicates Walter's liquid B [15] also termed as second order fluid. A remark should also be made that N can take positive and negative values or $N = 0$ (mass transfer is absent).

The existence of analytical solution for the current physical problem when convection, viscoelasticity and concentration are absent, is evident after Pop and Na [16], and the solution reads

$$f(\eta) = s + \frac{1 - e^{-\beta\eta}}{\beta}, \tag{3.7}$$

with $\beta = \frac{1}{2}(s + \sqrt{4 + 4M + s^2})$. Therefore, in the presence of convection, porosity, viscoelasticity, magnetic interaction, concentration and wall suction parameters, we adopt the form of physical solution (3.7), and hence write

$$f(\eta) = s + \frac{1 - e^{-\lambda\eta}}{\lambda},$$
$$\theta(\eta) = f'(\eta) = e^{-\lambda\eta},$$
$$\phi(\eta) = \theta(\eta) = e^{-\lambda\eta} \tag{3.8}$$

that enables us to unify the parameters γ, M, Λ and N appearing in Eq. (3.5) under a newly defined parameter $\Gamma = -\gamma - M + \Lambda(1 + N)$, which is hereafter named as the porous magneto-convection concentration parameter. Consequently, the energy equation and concentration equations in (3.5) produce the relations

$$\lambda^2 + \Pr(-1 - s\lambda + D_f\lambda^2) = 0,$$
$$\lambda^2 - Le(\Pr + \Pr s\lambda - Sr\,\lambda^2) = 0, \tag{3.9}$$

and further placing (3.8) into first of (3.5) gives the third equation

$$-1 + \Gamma + \lambda^2(1 - k) - s\lambda(1 + k\lambda^2) = 0. \tag{3.10}$$

Eqs. (3.9) and (3.10) can be rearranged as

$$\Gamma\,\Pr + \lambda^2(-1 + \Pr - k\lambda^2) = 0, \tag{3.11}$$

with the Prandtl number Pr given by

$$\Pr = \frac{1 + Le(-1 + Sr)}{D_f Le}. \tag{3.12}$$

It is worth to note here that Eq. (3.12) inserts a restriction on the Prandtl number, which is always positive. In another view, for a