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# A generalized thermoelasticity problem of an annular cylinder with temperature-dependent density and material properties

Ashraf M. Zenkour [a,*], Ibrahim A. Abbas [b,c]

[a] Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[b] Department of Mathematics, Faculty of Science and Arts-Khulais, King Abdulaziz University, Jeddah, Saudi Arabia
[c] Department of Mathematics, Faculty of Science, Sohag University, Sohag, Egypt

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ABSTRACT

In this paper, the problem of generalized thermoelasticity with one relaxation time for an infinite annular cylinder of temperature dependent physical properties is discussed. Both the inner and outer curved surfaces of the cylinder are considered stress free. The inner surface is subjected to decaying with time and temperature whereas the outer surface is maintained at a reference temperature. A finite element model is developed to derive the solution of the coupled non-linear partial differential governing equations. The transient solution can be evaluated directly from the model at any time. The numerical solution of displacement, temperature, and stresses is obtained inside the annulus for different forms of the temperature-dependent and temperature-independent material properties of the medium. The effect of temperature-dependent parameter and the relaxation time is investigated by different graphical plots.



## 1. Introduction

During last few decades considerable attention has been made for the theory of thermoelasticity in various fields of science, especially in engineering, biology, geophysics, plasma-physics, acoustics, etc. The theory of uncoupled thermoelasticity suffers from physical drawbacks that mechanical state of elastic body has no effect on the temperature and also the thermal signal propagates with infinite speed. It is found with true physical experiments that this theory contains two shortcomings which are not physically admissible. Based on the thermodynamic principles of irreversible processes, Biot [1] has introduced the theory of classical thermoelasticity. In this theory, the equation of motion is hyperbolic in nature, whereas the heat conduction equation is parabolic in nature. The theory predicts a finite speed for predominantly elastic disturbances but an infinite speed for predominantly thermal disturbances, which are coupled together. So, this theory removes the first drawback but shares the second defect of the uncoupled theory due to the presence of parabolic type heat conduction equation. Subsequently, efforts to eliminate this drawback have led to the generalized thermoelasticity theories in which the heat conduction equation is of hyperbolic type.

Lord and Shulman [2] have developed a theory of generalized thermoelasticity with one relaxation time by introducing the time derivative of the heat flux vector and a new constant acting as a relaxation time in classical Fourier law. The heat conduction equation of this theory is of wave type, ensuring finite speed of propagation of both the thermal wave and elastic wave and thus removes the second drawback of the uncoupled theory. This procedure is called the first generalization to the coupled theory of elasticity for isotropic homogeneous media. The corresponding theory is obtained by Dhaliwal and Sherief [3] by including the effect of anisotropic behavior and the presence of heat source. Due to complexity of the governing equations and mathematical difficulties associated with their solution, several simplifications have been used. Some authors neglect the relaxation time resulting in a parabolic system of partial differential equations. The solution of this system exhibits infinite speed of propagation of heat signals contradictory to physical observation. Other authors ignored the inertia effects in a coupled theory or neglected the coupling effect.

Because of the experimental evidence in support of the finiteness of the speed of propagation of a heat wave, generalized thermo-elasticity theories are more realistic than conventional thermo-elasticity theories in the treatment of practical problems involving very short time intervals and high heat fluxes. The second generalization to the coupled theory of elasticity is what is known as the theory of thermoelasticity with two relaxation times. A generalization of this inequality was proposed by Green and Laws [4]. Green and Lindsay [5] have obtained an explicit

* Corresponding author.
*E-mail address:* zenkour@gmail.com (A.M. Zenkour).

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**Nomenclature**

| | |
|---|---|
| $\lambda, \mu$ | Lamé elastic constants, |
| $\sigma_r, \sigma_\theta$ | radial and circumferential stresses, |
| $\varepsilon_r, \varepsilon_\theta$ | radial and circumferential strains, |
| $\varepsilon_{kk}$ | dilatation, |
| $T$ | absolute temperature, |
| $T_0$ | reference temperature, |
| $\alpha$ | coefficient of linear thermal expansion, |
| $\gamma = (3\lambda + 2\mu)\alpha$ | stress-temperature modulus, |
| $\rho$ | density, |
| $u$ | radial displacement, |
| $\tau_0$ | thermal relaxation time parameter, |
| $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial t}$ | Laplace operator, |
| $K$ | thermal conductivity, |
| $\kappa$ | thermal diffusivity |

version of the constitutive equations. This theory is also called the theory of temperature-rate dependent thermoelasticity and takes into consideration two relaxation times. Green and Naghdi [6–8] have also proposed three new formulations of thermoelasticity based on inequality of entropy balance.

Researches presented in the transient heat transfer analysis of the infinite cylinders so far are very limited and almost most of them have ignored the temperature-dependency of the material properties. The material properties of the medium are taken to be constant in most of the thermoelasticity problems. However, it is well known that the physical properties of engineering materials vary considerably with temperature. The temperature dependency of material properties should be taken into consideration to obtain more dependable solution of the problem of thermoelasticity. Noda [9] has showed that at high temperature the material properties no longer remain constant and the temperature dependency of material properties affects the thermo-mechanical behavior of the medium. The use of temperature-dependent physical properties in estimating the thermoelastic response of cylinders and tubes is assessed by Argeso and Eraslan [10]. In the context of generalized thermoelasticity theory with one relaxation parameter, Mukhopadhyay and Kumar [11] have investigated a problem of an infinitely long annular cylinder, whose material properties like modulus of elasticity and thermal conductivity linearly vary with temperature.

For many practical applications, it is sufficient to consider an infinite cylinder immersed in an infinite medium, with thermal surface resistance (contact resistance) at the boundary between cylinder and surrounding medium. The flow due to an oscillating cylinder is one of the most important and interesting problems of motion near oscillating walls. The rotational oscillations of an infinite cylindrical rod immersed in a Newtonian fluid represent an example of such case. In addition, the piles used in the foundations of and the long poles used in high-rise buildings are considered as infinitely-long cylinders. Some articles that deal with infinite cylinders are found in the literature (see, e.g., [12–16]).

This article presents the nonlinear transient thermal stress of temperature-dependent infinite cylinders subjected to a decaying with time thermal loading. To overcome this problem, the finite element solution procedure is adopted to extract the results from the highly nonlinear governing equations. Based on the assumptions of temperature-dependent and temperature-independent of the material properties, numerical results of temperature, displacement and stresses are compared. Moreover, the effect of relaxation time parameter is also studied.

## 2. Basic equations

Let us consider an infinitely long annular cylinder of inner radius $a$ and outer radius $b$ as shown in Fig. 1. The cylinder is subjected to a thermal field and its material properties and density are temperature-dependent. The cylindrical polar coordinates $(r, \theta, z)$ are used as coordinates of reference with positive direction on the $z$-axis along the axis of the cylinder. The axisymmetric plane strain problem is considered. The physical quantities are assumed to be functions of the radial coordinate $r$ and the time $t$ only.

Stress–strain relations may be reduced to express the basic stresses $\sigma_r$ and $\sigma_\theta$ in terms of their strains $\varepsilon_r$ and $\varepsilon_\theta$ and the temperature field $\overline{T}$ as

$$\sigma_r = (\lambda + 2\mu)\varepsilon_r + \lambda\varepsilon_\theta - \gamma\overline{T},$$
$$\sigma_\theta = \lambda\varepsilon_r + (\lambda + 2\mu)\varepsilon_\theta - \gamma\overline{T}, \tag{1}$$

where $\overline{T} = T - T_0$ and the strains $\varepsilon_r$ and $\varepsilon_\theta$ are given by

$$\varepsilon_r = \frac{\partial u}{\partial r}, \quad \varepsilon_\theta = \frac{u}{r}. \tag{2}$$

The equation of motion in cylindrical coordinates by neglecting the body force becomes

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}(\sigma_r - \sigma_\theta) = \rho\frac{\partial^2 u}{\partial t^2}. \tag{3}$$

The heat conduction equation in the absence of the heat source is given by

$$K\nabla^2\overline{T} + \frac{\partial K}{\partial r}\frac{\partial \overline{T}}{\partial r} = \frac{K}{\kappa}\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\overline{T} + \gamma T_0\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\varepsilon_{kk}. \tag{4}$$

The elastic properties $\lambda$ and $\mu$, the density $\rho$, the thermal conductivity $K$, and the stress-temperature coefficient $\gamma$ are all temperature-dependent. That is

$$\{\lambda, \mu, \rho, K, \gamma\}(\overline{T}) = \{\lambda_0, \mu_0, \rho_0, K_0, \gamma_0\}e^{-\beta\overline{T}/T_0}, \tag{5}$$

where $\lambda_0$, $\mu_0$, $\rho_0$, $K_0$ and $\gamma_0$ are constants and $\beta$ is a temperature parameter. It is to be noted that the case of temperature-independent is given when $\beta = 0$.

Therefore, the equation of motion and the heat conduction equation are given, respectively, by

$$(\lambda_0 + 2\mu_0)\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right) - \left[\frac{\beta}{T_0}\left((\lambda_0 + 2\mu_0)\frac{\partial u}{\partial r} + \lambda_0\frac{u}{r} - \gamma_0\overline{T}\right) + \gamma_0\right]\frac{\partial \overline{T}}{\partial r} = \rho_0\frac{\partial^2 u}{\partial t^2}, \tag{6}$$

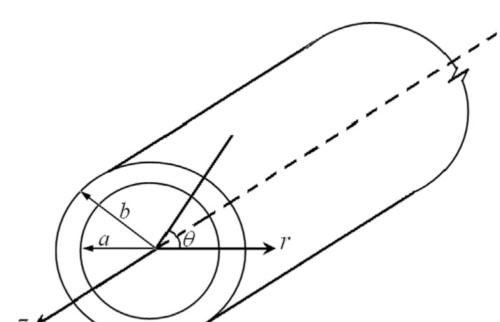


**Fig. 1.** Geometry of the infinite cylinder.