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An analytical solution for the estimation of the drawing force in three dimensional plate drawing processes

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ABSTRACT

An analytical solution for the estimation of the drawing force requested to perform a three dimensional drawing of a rectangular plate is presented.

It has been developed using the limit analysis technique, on the basis of a three-dimensional velocity field, under the main assumptions of constant friction and perfect plasticity.

To overcome the limitations given by these hypothesis, some extensions of the proposed upper bound model are presented, accounting for the material strain hardening and for the Coulomb friction between the plate and the die, thus avoiding the need to calibrate physically meaningless coefficients.

The effectiveness of the proposed results is demonstrated by comparison with both experimental results and new numerical solutions.

1. Introduction

The optimization of the plate drawing process has been slightly improved by the recent theoretical and numerical developments in mechanics: in the industrial practice, the determination of an appropriate working condition during the drawing process and the design of the optimum die shape and dimensions are still based on empirical knowledges, and on trial-and-error approaches. This is mainly due to the difficulties to develop simple techniques allowing the optimization of the die geometry as a function of the other process parameters (plate geometry, friction coefficient, area reduction, etc.), in order to minimize the production costs.

One of the main tools today available to study metal forming processes is the Finite Element Method (FEM). This numerical procedure models rather accurately the process mechanics as a function of the process parameters but, unfortunately, each different combination of possible die geometry, area reduction, or friction coefficient requires a single numerical analysis.

The optimization of a real industrial process implies parametric studies that involve many different values of the parameters (for example, different die geometries), and, using this method, several different models should be developed [8]. Moreover, the numerical simulation of metal forming processes requires in general a complex three-dimensional non-linear analysis, and a large amount of time must be spent to obtain a solution for each single set of parameters.

Moreover, if the numerical modelling involves not only the computation of the drawing stress as a function of the die geometry, but also other complex features, such as, for instance, the minimization of the residual stresses, or the prediction of defects into the final piece, one should adopt rather complicate constitutive models. For example, during the drawing process, the material is subjected to non-monotonic loadings, and the assumption of a kinematic strain hardening law is fundamental to the correct evaluation of the residual stress profiles in the final product [9]. Also the prediction of defects into the final piece requires a material modelling richer than that furnished by the von Mises isotropic plasticity, in order to simulate accurately the material response at very high hydrostatic pressures [11]. Obviously, a more complex constitutive modelling of the material requires higher computational costs.

For these reasons, even if the numerical analyses are probably the most powerful tool available to simulate metal forming processes, analytical design procedures, which allow (at least) an initial design of the process are very important. Moreover, it should be noted that the most adopted design procedures of metal forming processes in the engineering practice are still based on the limit analysis technique. This analytical method has been extensively employed to study the material flow through conical converging dies for rods and wires (see, for instance, [2,3,5]). Recently, the upper bound (UBET) solutions reported in these works, based on the assumption of perfect plasticity, have been extended to consider the strain hardening of the material during the drawing process [7,10]. Unfortunately, for dies with more complex geometry, only a few theoretical approaches to drawing or extrusion have been published (for instance, see [12] or [17]).

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This is mainly due to the difficulties to develop analytical expressions in a closed form to compute the drawing force taking into account the three-dimensional die shapes.

Rubio et al. [14] have recently published an analytical solution for two-dimensional (2D) plate drawing process assuming a perfect plastic material under plane strain condition. Their solution, which has been extended in [13] to consider the material strain hardening, is relatively simple. Unfortunately, since it has been developed for a 2D problem, it cannot be employed directly to optimize an industrial 3D plate drawing process.

In order to obtain estimations of the drawing force for three dimensional die shapes, the limit analysis techniques have been extended using the SERR (Spatial Elementary Rigid Region) technique (see, for instance, [15,16]). This mixed numerical–analytical technique is based on the subdivision of the deformation zone into tetrahedral blocks. The velocity of each block is then determined numerically, by imposing the compatibility conditions between the block interfaces. Then, this method does not furnish an expression for the drawing force in a closed form.

In this paper, a new analytical solution for the drawing force in three dimensional plate drawing processes is presented.

The proposed analytical model, based on the limit analysis technique, is obtained from a new three dimensional velocity field, assuming constant friction between die and plate, and perfect plasticity. The new solution is also extended considering Coulomb friction and accounting for the material strain hardening.

The proposed solutions is finally validated by comparison with new FEM solutions.

2. The velocity-based analytical solution

The upper bound solution here presented allows the estimation of the drawing stress σ_{zd} necessary to perform a reduction of a rectangular plate, with initial dimensions equal to $2b_i \times 2h_i$ to the final dimensions $2b_f \times 2h_f$.

Let assume that the initial rectangular shapes of the die and of the workpiece are *similar*, i.e., that the ratio between the width and the height of the die at the entrance is equal to that of the plate. If this condition is not respected, the plate becomes initially in contact only with two faces of the die (top-bottom or left-right), and the reduction is initially limited to one dimension (height, or width), until all the lateral surface of the plate is in contact with the die. It should be noted that the condition of different dimensional ratios between plate and die is not common in the industrial practice. In fact, the nonhomogeneous deformation of the plate faces initially free causes high friction stresses, that provoke a quick wearing of the die surfaces and a worst quality of the final product (possible surface defects).

Under this assumption, the die geometry can be fully described as a function of the plate initial and final dimensions, and of a single inlet reduction angle (for example, as a function of height reduction angle α).

The problem here studied has two symmetry planes, and then, only a quarter of the plate is considered.

Initially, the material is assumed to be rigid perfectly plastic, and the shear along the contact interfaces between die and plate is modelled assuming a constant friction. This model will be extended in the next sections, considering a Coulomb friction model to describe the interactions between die and plate, and accounting for the strain hardening of the material.

2.1. The assumed three-dimensional velocity field

An upper bound analytical approach is based on a *postulated* velocity field, that must respect the boundary conditions, i.e., for this problem, the plate symmetry planes, and the constraints given by the contact with the die surfaces.

Here we assume that the plate in the plastic zone can be subdivided into eight blocks, in which the material is subjected to rigid body motion.

The blocks subdivision here adopted is reported in Fig. 1. This particular geometry of the blocks assures two main features, both of them related to the directions of the velocities of the blocks themselves. In particular, the directions of both the absolute velocity of each block and that of the relative velocity between two neighboring blocks are *a priori* known.

The first feature derives from the external boundary conditions: the plate has two symmetry planes (*yz* and *xz* in Fig. 2a), and it is constrained by the two planes π_1 and π_2 (Fig. 2b), on which it is in contact with the die surfaces. Hence, the direction of the absolute velocity of each rigid block can be determined *a priori*, as a function of the interactions of the block itself with these planes (Fig. 3a).

The second feature results from the compatibility condition between each couple of neighboring blocks: the only admissible *relative* motion between two neighboring blocks is a slip along the face that they share (the discontinuity surface). This condition implies that the relative velocity between each couple of contiguous blocks must lie on the plane of their discontinuity surface (Fig. 3b). In order to respect this geometric constraint, two different features have been adopted for the block geometry. First, the block subdivision has been obtained in such a way that the absolute velocities of each couple of neighboring blocks are coplanar. This obviously implies that also their relative velocity lies on the plane of their absolute velocities (the plane of the velocities in Fig. 3b). Second, the discontinuity surface (i.e., their shared face) is constructed to be always normal to this velocity plane. In this way, for each couple of rigid blocks, the direction of their relative velocity is *initially* known, since it is given by the intersection between these two planes.

The geometry of each block can be described as a function of the die geometry, and of a distance η from the plate cross-section at the die exit and the point *B*, placed on the drawing axis. The die geometry is initially known as a function of the plate initial and final dimensions, and of the reduction angle α on the plane *xz*, whilst η will be determined in order to optimize the upper bound, i.e., to minimize the computed drawing stress. Both the die and the blocks geometry can be easily determined from Fig. 4: all the

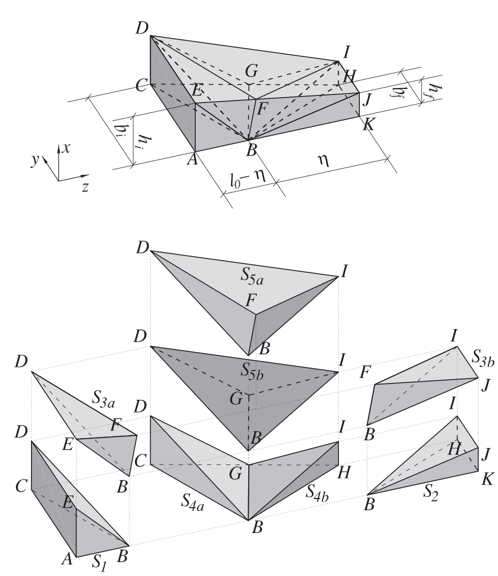

Fig. 1. Assumed blocks subdivision for the plastic zone.