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Helical buckling of a thin rod with connectors constrained in a cylinder

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ABSTRACT

Kinds of tubular columns with connectors are widely used in petroleum exploration and development field. In previous research, tubular columns are usually treated as homogeneous thin elastic rods constrained in a cylinder and little attention has been paid to the effect of connectors.

This paper builds an analytical model to describe the comprehensive helical buckling behaviors of a rod with connectors. The three-dimensional buckling problem is divided into two lateral buckling problems in two mutual-perpendicular planes and the buckling solutions are obtained by solving the two lateral buckling equations.

There are three contact cases between the rod and the cylinder, namely the no contact, point contact and wrap contact. The deflection curves of a rod in three cases and critical conditions between different contact cases are calculated in this paper. The effects of connectors on the length of the contact portion, bending stress and contact force of the rod are analyzed. The results indicate that the critical axial compressions only depend on the ratio of rod/cylinder radial clearance to connector/cylinder radial clearance and the effect of connectors reaches the maximum value under the critical condition from the no contact case to the point contact case. The comparison between the buckling solutions with and without connectors shows that the effects of connectors cannot be ignored in buckling analyses.

1. Introduction

Tubular columns such as drill string, casing, and tubing play an important role in oil exploration and development. Taking the drill string, for example, it transmits mechanical power (torque and rotation), hydraulic power (pressure and flow rate) and weight of the drill string to the bit to break underground rocks [1] as shown in Fig. 1. The outer diameters of these tubular columns are usually smaller than 0.5 m, but they extend for thousands of meters or more. The tubular columns have quite small radial displacements under the constraint of the wellbore of which the diameter is usually no larger than 1 m. Therefore, these tubular columns are usually seen as thin elastic rods and the wellbore is taken as a cylinder which constrains the radial displacements of rods.

Euler solved the lateral buckling problem of a thin rod under compression for the first time. But the solution was not applicable for constrained buckling problems of rods. Therefore, the following research was focused more on the buckling behaviors of a rod constrained in a cylinder.

Lubinski [2] studied the helical buckling of a weightless rod by virtual work principle and derived the relationship between the axial compression and the deflections of rods. Paslay and Bogy [3] analyzed the stability of a rod constrained to an inclined cylinder by virtual work principle and gave the critical load for sinusoidal buckling. Mitchell [4–7], Gao [8–10][] and Liu [11] deduced the general differential equations of buckling of a rod and solved the buckling problems while considering the effects of weight, friction and so on. These results have been widely used in engineering practice.

However, all these studies are all under the assumption that the rod is homogeneous and in continuous contact with the cylinder. In practical applications, connectors distribute discretely along the rod and the diameters of connectors are larger than that of the rod. As a result, some portions of the rod lose contact with the cylinder for the existence of connectors. That is to say, the assumption of the continuous contact between the rod and the cylinder does not hold in this case. The effects of connectors should not be neglected in buckling analyses. As to our knowledge, only several studies consider the effects of connectors. Mitchell assumed that only connectors were in contact with the cylinder and solved the helical buckling of a weightless rod [12] and the sinusoidal buckling of a rod with weight [13,14].

In fact, with the increase of the axial compression, the rod body between two adjacent connectors will bend and touch the cylinder. Lubinski [15] studied the two-dimensional buckling of a rod with connectors in tension in no contact, point contact and wrap contact cases, and Paslay [16] studied the two-dimensional buckling of a rod with connectors in compression in no contact,

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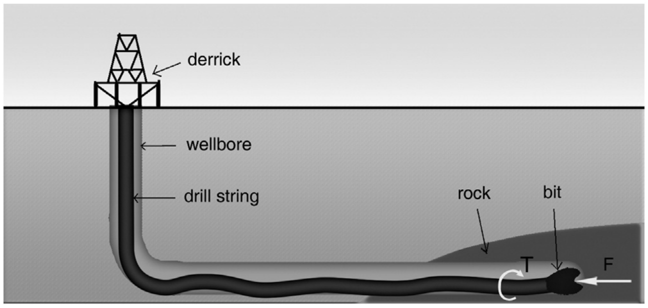

Fig. 1. Drill string in the wellbore.

point contact and wrap contact cases. However, little research has considered the three-dimensional buckling of a rod which is in contact with the cylinder.

In this paper, the no contact, point contact and wrap contact cases of rod buckling are depicted by a new buckling model. The deflection curves of the rod in the three cases are obtained and the transition conditions between different contact cases of the rod are given. A detailed analysis is made about the effects of connectors.

2. Buckling model of a thin rod with connectors

2.1. Background

As is shown in Fig. 1, the underground rock is destroyed by the axial compression F and torque T applied to the bit. Therefore, the drill string is in axial compression in the horizontal wellbore when the bit is working. Since the drill string and the wellbore can be respectively taken as a thin rod and a cylinder, buckling behaviors of the drill string in the horizontal wellbore can be obtained by solving the buckling model of a compressed thin rod in a horizontal cylinder.

For a compressed rod in a horizontal cylinder, buckling initiates in a sinusoidal configuration in which the rod snakes along the lower surface of the cylinder. With the increase of the axial compression, the rod achieves the helical buckling state, in which the rod forms a helix spiraling around the inner surface of the cylinder [17]. For a weightless rod, the sinusoidal buckling stage does not exist and we only consider the helical buckling stage.

The helical buckling of a rod with connectors is shown in Fig. 2. The rod is constrained in the cylinder and connectors are contact with the inner surface of the cylinder. When there are no connectors, the rod forms a helix and is continuous contact with the cylinder. However, the rod will lose contact with the cylinder for the existence of connectors.

When an axial compression F is applied at the ends of the rod, the problem becomes complicated for the axial compression has an opposite effect compared with the connectors and makes the rod tend to be contact with the cylinder. Finally, there are three cases of contact state, caused by the connectors and the axial compression, between the rod and the cylinder.

For a low axial compression, the rod suspends between connectors and is not in contact with the cylinder, provided that the parameters of connectors are given in advance. As the axial compression increases, the rod is in contact with the cylinder at a single point, or even in continuous contact with the cylinder. We call these three cases the no contact case, the point contact case and the wrap contact case.

The geometric parameters involved in this paper include the diameter of the cylinder D_y, the diameter of the connectors D_c and the diameter of the rod D_d as seen in Fig. 3. Then the radial clearance between the connector and the cylinder is

$$r_c = \frac{D_y - D_c}{2} \tag{1}$$

and the radial clearance between the rod and the cylinder is

$$r_d = \frac{D_y - D_d}{2} \tag{2}$$

In the following analysis we will see that the parameters r_c and r_d play an important role in the buckling of the rod.

2.2. Assumption

The investigation in this paper is based on the following assumptions:

1. The axis of the cylinder is straight and horizontal.
2. The connectors are taken as points of support and contact with the cylinder.
3. Elastic theory is satisfied.
4. Friction, torque and weight are neglected.
5. Every portion of the rod between two adjacent connectors has the same deflection curve.

2.3. No contact case

Without loss of generality, we only discuss the deflection between the two adjacent connectors A and B as shown in Fig. 2. The rod buckling problem is three-dimensional and it is divided into two lateral deflections of a beam in the y–z and x–z planes as seen in Fig. 4. The buckling solutions in the y–z and x–z planes are derived separately, using the beam-columns model with two supports in Appendix A.

The lateral displacements of connectors A and B in the y–z plane and x–z plane are calculated by

$$\begin{aligned} h_{ay} &= r_c \\ h_{by} &= r_c \cos \alpha_B \\ h_{ax} &= 0 \\ h_{bx} &= r_c \sin \alpha_B \end{aligned} \tag{3}$$

where α_B denotes the angular displacement between the connectors A and B.

With Eqs. (A.3) and (A.4), we obtain the relationship between rotation angles and bending moments at two connectors

$$\begin{aligned} \theta_{ay} &= \frac{M_{ay}L}{3EI}\psi(u) + \frac{M_{by}L}{6EI}\phi(u) + \frac{h_{by}-h_{ay}}{L} \\ \theta_{by} &= \frac{M_{by}L}{3EI}\psi(u) + \frac{M_{ay}L}{6EI}\phi(u) - \frac{h_{by}-h_{ay}}{L} \end{aligned} \tag{4}$$

$$\begin{aligned} \theta_{ax} &= \frac{M_{ax}L}{3EI}\psi(u) + \frac{M_{bx}L}{6EI}\phi(u) - \frac{h_{bx}-h_{ax}}{L} \\ \theta_{bx} &= \frac{M_{bx}L}{3EI}\psi(u) + \frac{M_{ax}L}{6EI}\phi(u) + \frac{h_{bx}-h_{ax}}{L} \end{aligned} \tag{5}$$

Assumption (5) indicates that the deflection curve between connectors A and B in the o–x–y–z coordinate system is identical with that between connectors B and C in the o–x'–y'–z coordinate system. Therefore, the bending moments and slopes of the deflection curves at the connector A in the o–x–y–z coordinate system are identical with that at the connector B in the o–x'–y'–z coordinate system. Thus, we obtain

$$\begin{pmatrix} M_{ax} \\ M_{ay} \end{pmatrix} = \begin{pmatrix} \cos \alpha_B & \sin \alpha_B \\ -\sin \alpha_B & \cos \alpha_B \end{pmatrix} \begin{pmatrix} M_{bx} \\ M_{by} \end{pmatrix} \tag{6}$$

$$\begin{pmatrix} \theta_{ax} \\ \theta_{ay} \end{pmatrix} = \begin{pmatrix} \cos \alpha_B & \sin \alpha_B \\ -\sin \alpha_B & \cos \alpha_B \end{pmatrix} \begin{pmatrix} -\theta_{bx} \\ -\theta_{by} \end{pmatrix} \tag{7}$$