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A novel approach for determining material constitutive parameters for a wide range of triaxiality under plane strain loading conditions

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ABSTRACT

This paper focuses on material constitutive models for plane strain applications such as orthogonal cutting. It presents a novel approach to determine the material model parameters which utilizes a new, adjustable stress state specimen, Digital Image Correlation (DIC) measurements, and an inverse method for parameter identification. The proposed double-notched specimen is purposely designed to allow the identification of damage and fracture parameters for the plane strain condition. The corresponding equivalent plastic strains at different stages of deformation and damage were calculated from DIC measurements. The elastic modulus and yield surface were obtained using a numerical optimization based inverse method. Ultimately, the fracture locus was obtained and the parameters in the Rice and the reduced form of the Johnson Cook (JC) damage models were identified. The model validation is also provided.

1. Introduction—a need for a new approach

A material constitutive law capable of describing all phases of deformation including damage and fracture is essential in many applications that require predictions. Modeling real life operations such as manufacturing processes depends very much on the applicability and accuracy of the relations used to describe the mechanics of the material. A material model which has been effectively used in one particular process may not be the case for the others due to alteration in loading condition and/or deformation phase change.

Consider for example two functionally close but different loading mechanisms, stamping and forming on the one hand, and machining on the other hand. Numerical simulations which provide precise predictions of material plasticity and ductility limitation have revolutionized the mold design in manufacturing processes such as stamping and forming. In contrast, the available material models are still insufficient to provide useful information to guide the improvement of the design of cutting tools and process optimization.

Astakhov defined the metal cutting process as the purposeful fracture of the layer of the work material [1,2]. Moreover, he pointed out that the most common model of metal cutting which does not include fracture is inadequate to any real machining process [3]. Analyzing the existing materials models used in metal cutting modeling including FEA, Astakhov showed [4] that as even the simplest cutting, known as orthogonal cutting, involves a triaxial state of stress which can be altered/optimized to achieve the minimum energy required for cutting and thus the greatest tool life, and machined surface integrity and thus the highest process efficiency. Moreover, he pointed out that any further advance in metal cutting modeling can be achieved if a model that includes fracture characterization of the work material is developed.

To describe the material behavior where large deformation may cause damage and separation, the models must go beyond the ductility limit to include damage and fracture. Furthermore, the models must capture the material behavior under the conditions where deformation and fracture take place.

The objective of this paper is to develop a new approach to material constitutive modeling suitable for the use where the work material is in a plane strain condition and its response can be modeled with 2D simulations. The proposed approach includes a methodology for the determination of the elastic modulus, yield surface and fracture locus in material characterization for the model. The overall effectiveness of the current approach in modeling the orthogonal machining has been demonstrated by the authors earlier in a preliminary study [5].

2. Considerations for constitutive models

2.1. Fracture criteria in ductile metals

Material separation is the result of a complex physical process which occurs at the microscale. At the macroscale, the only

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variables that control fracture are current values of component stress and strain tensors and their histories [6]. Fracture criteria are formulated based on the stress, strain, and their combinations. Among them, a constant equivalent strain criterion is often used which assumes material fracture occurs when the equivalent plastic strain $\bar{\varepsilon}$ reaches a certain predefined value $\bar{\varepsilon}_f$. Throughout the history of fracture studies, it was observed that fracture could also occur due to material strength limitations on certain stress components. For example, maximum shear stress criterion predicts fracture on a plane where its shear stress component τ exceeds a critical value τ_{max}. Nevertheless, the suitability of such criteria is limited to particular engineering problems, and thus for many applications, a more general approach is needed.

The mechanism of ductile fracture of metals is identified as the formation, growth, and coalescence of microscopic voids. The growth rate of micro-voids under combined state of stress that includes the normal and shear stresses has been investigated by a number of authors. McClintock [7] studied the growth of voids of a cylindrical shape and concluded that the ratio of the hydrostatic stress (σ_m) to the effective stress ($\bar{\sigma}$), also known as the stress triaxiality state parameter (η), is a predominant parameter in damage formulation. A similar work was conducted by Rice and Tracey [8] who investigated stress triaxiality effects on the micro-voids growth of a spherical shape and observed that the growth rate is significantly affected by the superposition of hydrostatic tension on a remotely uniform plastic deformation field. For both the cylindrical and spherical voids shapes, Rice and Tracey indicated that moderate and high stress triaxiality leads to amplification of the relative void growth rates over imposed strain rates by a factor depending exponentially on the mean normal stress.

Atkins [9] studied fracture in bulk and shear forming processes and stated that the hydrostatic stress state is an important parameter which seems to have a predominant effect on the volume change of the voids. Although such growth theories consider only a volume change of a particular void shape, the volume changing contribution to the void growth is found to overwhelm the shape changing part when the mean remote normal stress is large [8,9]. Such a justification is necessary to assume a symmetrical growth throughout the deformation process.

It was pointed out that the material ductile fracture may be affected not only by the state of hydrostatic stress but also by the path under which this deformation was developed [9,10]. The process of fracture is strictly path-dependent and the fracture strain in one process may differ from another. For this reason the damage function defined by Eq. (1) is always represented in an integral form of the effective strain ($\bar{\varepsilon}$) path and weighted by an “arbitrary function *f*”. The damage is assumed to initiate when a damage indicator *d* reaches a certain predefined critical value

$$d = \int_0^{\bar{\varepsilon}_f} f(state\ of\ stress)\,d\bar{\varepsilon} \tag{1}$$

Studies have shown that the state of stress in Eq. (1) is not limited only to the state of the hydrostatic stress (or equivalently, stress triaxiality). Ductile and brittle metals may also rupture due to shear stresses. For example, Leppin et al. [11] have combined the ductile and shear fracture mechanisms and postulated that fracture occurs when the maximum value of the two components reaches unity. In addition to the triaxiality state parameter, the authors included the ratio of the maximum shear stress and effective stress in crashworthiness simulations.

Wilkins et al. [12] proposed a cumulative fracture criterion in which a weighting function *f* is defined such that $f = w_1 w_2$. The first term w_1 is a function of the hydrostatic pressure whereas w_2 represents the effect of the deviatoric stress tensor. The effect of deviatoric stress tensor principle was introduced in many recent studies as the second fracture dependent state parameter [13,14].

The deviatoric state parameter has been formulated as a function of the third deviatoric invariant and effective stress ratio. Wierzbicki and Xue [14] suggested a new fracture criterion and assumed an accumulated equivalent plastic strain model similar to Eq. (1). The model damage function involves the two stress state dependent parameters, the stress triaxiality and the deviatoric state parameter. It accurately explains most, if not all, experimental observations. Another significant advantage is the relative simplicity of its calibration [6].

2.2. Stress state parameterization

Bai and Wierzbicki [13] showed that two parameters may be used to describe the material state of stress, the stress triaxiality state parameter (η) and the deviatoric state parameter (ξ).

The stress triaxiality state reflects the effect of the mean stress (σ_m) which is equivalent to the spherical part of the stress tensor. The stress triaxiality state is defined by a dimensionless parameter:

$$\eta = \frac{\sigma_m}{\bar{\sigma}} \tag{2}$$

and is considered as an important factor in formulating ductile fracture models in the literature [15,11,14,16,6].

Another parameter of the stress state, known as the normalized third deviatoric invariant, affects a material's ductility, and thus affects its fracture strain [16,6,17]. This parameter considers the influence of some combination of the deviatoric part of the stress tensor which relates to the so-called Lode angle (θ). The deviatoric state parameter and the Lode angle are defined as follows:

$$\xi = \frac{27}{2}\frac{J_3}{\bar{\sigma}^3} = \cos(3\theta) \tag{3}$$

$$\bar{\theta} = 1 - \frac{6\theta}{\pi} \tag{4}$$

where J_3 is the third invariant of the deviatoric tensor and $\bar{\theta}$ is the normalized Lode angle. The deviatoric state parameter (ξ) and $\bar{\theta}$ both have a valid range of [−1,1].

2.3. Fracture locus under special loading conditions

Bai and Wierzbicki [13] introduced an asymmetric fracture locus in the space of equivalent fracture strain, stress triaxiality state parameter and the Lode angle parameter. The authors proposed a fracture model based on the experimental observations of the material fracture dependency on the stress state parameters.

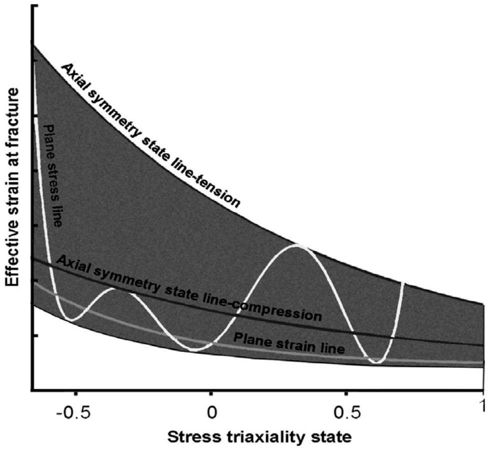

Fig. 1. Typical fracture locus for a number of spacial loading conditions in strain–triaxiality space.