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Large deflection and stresses in variable stiffness composite laminates with curvilinear fibres

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ABSTRACT

Large deflection and stresses of variable stiffness composite laminated (VSCL) plates with curvilinear fibres are studied. In each ply of these plates, the fibre-orientation angle changes linearly with respect to the horizontal coordinate. The manufacturing restrictions that exist regarding the fibre curvatures in this type of laminates are taken into account. To carry out the analyses, a new *p*-version finite element, which follows third-order shear deformation theory, is employed. Deflections, normal and transverse (with constitutive and equilibrium equations) stresses are determined as functions of tow-orientation angles in the non-linear regime.

1. Introduction

The idea of spatially steering fibre paths in the plane of a composite laminate was launched in the early 90s [1–3], introducing a new approach to develop variable stiffness composite laminates-VSCL. This tailoring concept has advantages in comparison with other ways of varying the stiffness of composite laminates. One of them is that the stiffness properties vary continuously with the membrane coordinates, in opposition to terminating plies at different locations (an alternative way of achieving variable stiffness). The latter option leads to abrupt changes in panel thickness and to ply drop-offs, which produce stress concentration and out-of-plane (interlaminar) stresses [4]. VSCL plates with curvilinear fibres have many potential applications in aerospace and naval structures [5]. For example, in an aircraft fuselage some regions dominated by bending are adjacent to regions where shear deformation mostly affects the response; therefore, plies with fibres aligned along a certain axis should alter to a forty-five degree orientation with relation to the same axis [5]. A VSCL with curvilinear fibres can answer this problem without introducing discontinuities.

With the expansion of the idea of using VSCLs with curvilinear fibres (from now on, designated simply as VSCL), investigators [3,6,7] discussed the novelties of VSCLs in buckling, failure, stresses, and static deformation when the plates are subjected to in-plane uniaxial loading and end shortening. These novelties are due to the ability of VSCLs to re-distribute stresses via the spatial variation of fibre angles. After these achievements, Tatting and Gürdal developed a design tool, where a solver designated as STAGS was integrated with a laminate design software [8,9]. With the help of the software developed, Tatting and Gürdal succeeded to continue analysing buckling, prebuckling, deformation, and stress distributions in a VSCL plate subjected to in-plane loading (with emphasis in plates with a hole, because of the usefulness of VSCL in distributing the stresses created around the hole). Recent works show the continuing interest of researchers in uncovering unknown aspects of VSCLs with curvilinear fibres [10–14]. Senocak and Tanriover [10] studied VSCL plates subjected to in-plane loading, using Galerkin method. In Refs. [12–14] buckling, failure and interlaminar stresses of VSCL plates were investigated using ABAQUS finite element software. The studies included normal deflection and normal stresses of the plate. To the best of the authors' knowledge, an extensive study portraying deflections and stresses on VSCLs and taking into account geometrical non-linearity does not exist.

On the other hand, there are several publications addressing deflections and stresses aspects on CSCL (constant stiffness composite laminated) plates, i.e. in laminates with straight fibres. One can find 3-D elasticity exact solution for static analysis of multi-layered (CSCL) plates in Ref. [15]. Reference [16] showed experimental and classical plate theory (CPT) results on the deflection of CSCL plates in the non-linear regime. Putcha and Reddy [17]

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presented a refined mixed shear flexible finite element for the non-linear analysis of CSCL plates. Zhang and Kim [18] offered a 3-node triangular element for linear and geometrically non-linear analysis of deflections of laminated composite plates. A very comprehensive review including geometric non-linear finite element analysis of CSCL plates can be found in Ref. [19]. A particular mixed-enhanced finite element for linear bending analysis of CSCL plates based on First-Order Shear Deformation Theory (FSDT) was presented in Refs. [20–22]. There, even though FSDT was used, transverse shear stresses in composite plates were quite well computed.

The current tailoring concept may allow one to implement the best fibre angles for minimising deflections and/or to find VSCLs with lower stresses, for the same constitutive materials, overall panel dimensions and loadings. In the present paper, the authors aim at computing large deflections (taking geometric non-linearity into account) and stresses of diverse VSCL plates with curvilinear fibres and showing that the fibre variation may indeed be used to reduce deflections and stresses in some static loadings. To carry out the analysis, a new *p*-version finite element that follows Third-Order Shear Deformation Theory (TSDT) [23–25] is developed. Earlier, a linear version of this element has been applied to determine the linear modes of vibration of VSCL [26]. The new non-linear element is employed in diverse test cases on CSCL taken from the literature. We found, as is typical in *p*-elements, that accurate results are computed with a relatively small number of degrees of freedom. Deflections, normal stresses and shear stresses are determined as functions of tow-orientation angles in demonstrative examples. Shear transverse stresses are calculated with both constitutive and equilibrium equations. The effects of geometric non-linearity on the deflection and stresses of VSCL are shown. The model presented allows to continue searching for advantages of VSCL plates and to better understand the behaviour of these plates in different loading conditions.

2. Model for VSCL plate with curvilinear fibres

2.1. Fibre orientation

We study a symmetric laminate with length a and width b, and a Cartesian coordinate system is used with x and y rectangular coordinate axis on the midplane of the undeformed laminated plate (Fig. 1). The orientation of the reference fibre path in each layer of the VSCL plate is given by

$$\theta(x) = 2(T_1 - T_0)\left|x\right|/a + T_0 \tag{1}$$

in which T_0 and T_1 are, respectively, the path orientations at the centre and at distance $a/2$ from the centre of the rectangular lamina (Fig. 2). To produce other fibre paths in the lamina, there are generally two techniques: parallel and shifted method [3]. In the parallel method, each curvilinear fibre path is parallel to the other fibre paths, which changes the tow-orientation in different fibres and leads to more difficult computations. But in the shifted method, fibre paths have the same pattern, i.e. the tow orientation is equal in all fibres of the lamina, making the computations easier. In this paper, shifted fibres are considered, producing fibre paths by shifting the reference fibre path along the y-direction. To find further details on the fibre orientation, one can see Ref. [3]. Here, a VSCL $[\langle T_0,T_1\rangle^{(1)},\langle T_0,T_1\rangle^{(2)}]_{sym}$ plate exemplifies a four-layer symmetric laminated plate with fibre angles at first and forth layer as $\langle T_0,T_1\rangle^{(1)}$ and at second and third layer as $\langle T_0,T_1\rangle^{(2)}$.

A problem that arises in the fabrication of VSCLs is fibre kinking [3]. To avoid this, the orientation of the fibres should be chosen from an appropriate range, imposing a manufacturing constraint. Hence, the curvature of fibres is limited and should not be larger than a definite value [3,26]. The hatched area in the figures of this paper (like in Fig. 5) indicates fibre angles for which a VSCL plate cannot be built due to manufacturing limitations [3,26]. If the curvature of the fibre orientation is more than a defined value (here 3.28 m^{-1} after Ref. [3]; this value can be indicated by the manufacturer), significant gaps and overlaps appear in VSCL plate and make it uneven.

2.2. Equations of equilibrium

A new *p*-version finite element with hierarchical basis functions is now presented. This element follows a TSDT, because this theory allows one to consider shear deformation, and is hence more accurate than classical plate theory. Furthermore, in comparison with FSDT, it leads to more reasonable transverse shear stresses and does not require a shear correction factor [23–25]. In fact, although a mixed finite element formulation that considers the out-of-plane shear stresses as primary variables and that is based on a FSDT without shear correction factor is proposed in Ref. [27], FSDT generally implies the use of shear correction factors. The finite element here introduced, is an extension of the one presented in Ref. [26]; in particular, the displacement field is the same:

$$u(x,y,z) = u^0(x,y) + z\phi_x(x,y) - c_1 z^3(\phi_x(x,y) + \partial w^0(x,y)/\partial x)$$
$$v(x,y,z) = v^0(x,y) + z\phi_y(x,y) - c_1 z^3(\phi_y(x,y) + \partial w^0(x,y)/\partial y)$$
$$w(x,y) = w^0(x,y) \tag{2}$$

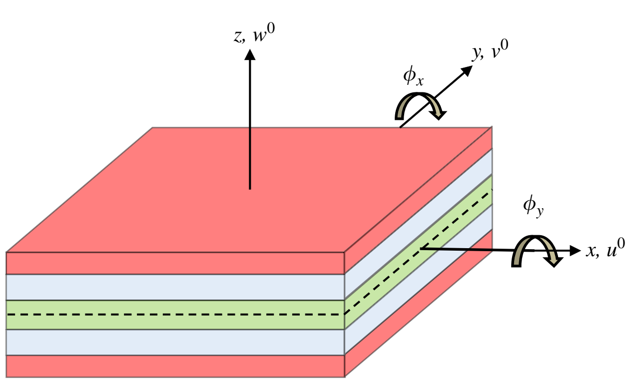

Fig. 1. Displacements and rotations in a symmetric laminate with Cartesian coordinates.

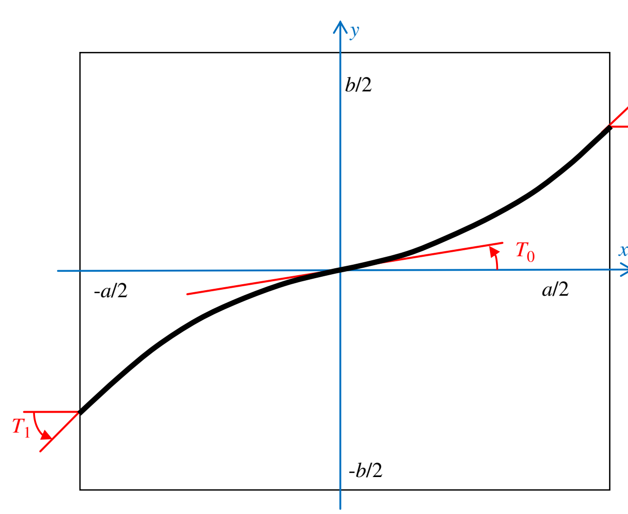

Fig. 2. Fibre orientation in a variable stiffness lamina.