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Large amplitude vibration and parametric instability of inextensional beams on the elastic foundation

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ABSTRACT

The nonlinear vibration and parametric instability of the inextensional beam on the elastic foundation are investigated. Considering the inextensional condition and second-order moment of the subgrade reaction, the motion equation of the beam on the elastic foundation is obtained via the Hamilton principle. Then the Galerkin method is applied to obtain the discretized model. The periodic motion is examined by means of the continuation method. Whereas, the numerical simulation is performed to study the nonperiodic motion, particular attention is placed on the nonlinear response of two resonant beams and the parametric instability. Finally, the effects of the boundary conditions and foundation models on the nonlinear vibration of the beams are discussed.

1. Introduction

The beams resting on the elastic foundations can model many soil-structure interaction (SSI) problems, and have wide application in many engineering fields. Therefore, it is important to investigate the vibration problem of the beam on the elastic foundation.

Lai et al. [1] studied the natural frequencies and mode shapes of the beam on the elastic foundation. Eisenberger [2] studied the vibration problems of the beam on one- and two-parameter foundations. Yokoyama [3] determined the vibration characteristics of the uniform Timoshenko beam on the two-parameter elastic foundation. Thambiratnam and Zhuge [4] investigated the free vibration character of the beam on the elastic foundation. Morfidis [5] presented the analysis on the natural vibration of the Timoshenko beam on the Kerr foundation. On the other hand, some studies focused on the dynamic response of the beam on the viscoelastic or tensionless foundation [6–8].

As a very important topic in structural dynamics, the moving load problem of the beam on the foundation is of practical importance in civil engineering. Kim and Cho [9] investigated the dynamic response and stability of the infinite beam on elastic foundation subjected to the moving loads. Zheng et al. [10] studied the stability of the axially compressed beam resting on a viscoelastic foundation with a moving vehicle. Chen and Huang [11] examined the dynamic characteristics of a railway subjected to a moving load.

However, very few studies concerned with the nonlinear vibration of the beam on the foundation. Pellicano and Mastroddi [12] examined the nonlinear vibration of the beam resting on the nonlinear foundation. Coskun and Engin [13] studied the nonlinear response of the beam on the nonlinear foundation. Zhu and Leung [14] studied the nonlinear vibration of the beam on two-parameter foundation. Moreover, Nayfeh and Lacarbonara [15] investigated the nonlinear response of the beam on the nonlinear foundation. Recently, considering the second order moment, Wang et al. [16] presented a refined model of the beam on the elastic foundation, and the quadratic nonlinear term and parametric excitation term are included. Based on this model, Wang and his co-workers [17,18] investigated the nonlinear interaction of the beam on the elastic foundation. Their results showed that the second-order participation of the SSI may destroy the conservation character of the beam-foundation system. However, this study neglected the effects of the parametric excitation term. In fact, this term may result in very complex dynamics of the structure [19–21].

In this study, the nonlinear vibration of the inextensional beam on the elastic foundation is investigated. Applying the Hamilton's principle, the planar model of the inextensional beam is derived, and it is discretized by the Galerkin method. The

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periodic and nonperiodic motion of the inextensional beam are examined by numerical results. Moreover, the nonlinear responses of the hinged-free beam and free-free beam on the elastic foundation are examined.

2. Continuum model

In this study, we consider a beam on elastic foundation with length L, width b and height h, as shown in Fig. 1. The in-plane displacements are denoted by $u(x,t)$ and $v(x,t)$ along the x and y directions. Moreover, the beam is assumed to subject to a uniformly distributed harmonic excitation: $p(x,t)=P\cos\omega t$, with P and ω being the amplitude and frequency.

2.1. Axial strain and inextensional condition

Referring to Fig. 1, the axial elongation of the element is $e=\sqrt{(1+u')^2+v'^2}-1$, where the prime indicates the derivative with respect to x. Obviously, if we assume that the plane sections are perpendicular to the deformed axis after deformation, the axial strain of the beam is $\varepsilon=e-y\kappa$, where $\kappa=\mathrm{d}\theta/\mathrm{d}x$ and θ is the rotation angle of the cross-section. From the expression of the axial elongation of the element beam, we can obtain:

$$\cos\theta=\frac{1+u'}{1+e},\quad \sin\theta=\frac{v'}{1+e}.$$

In this case, if we assume that u and v are small, we can obtain:

$$\theta=\arctan\frac{v'}{1+u'}=v'-u'v'-\frac{1}{3}v'^3+\cdots,$$

[22] , where we only keep up to cubic term in the expression of the rotation angle of the cross section. If we also assume that the beam is inextensional ($e=0$), we can obtain

$$u'=\sqrt{1-v'^2}-1\approx-\frac{v'^2}{2}\quad\text{and}\quad \kappa=v''+\frac{1}{2}v''v'^2. \tag{1}$$

2.2. Variational formulation

The motion equation of the beam on the elastic foundation can be obtained by using the extended Hamilton's principle, which can be expressed as

$$\delta\int_{t_1}^{t_2}(T-U)\,\mathrm{d}t+\int_{t_1}^{t_2}\delta W\,\mathrm{d}t=0, \tag{2}$$

where T is the kinetic energy of the beam; U is the strain energy of the beam; δW is the variation of the virtual work of non-conservative forces; and δ is the first variation.

If we consider the effects of the rotary inertia, the kinetic energy T can be expressed as

$$T=\int_0^L\frac{1}{2}(m\dot{v}^2+j\dot{\theta}^2)\,\mathrm{d}x, \tag{3}$$

where the dot indicates the derivative with respect to t; $j=\int_A\varrho y^2\,\mathrm{d}A$ is the rotary inertia; θ is the rotation angle of the cross-section; A and ϱ are the area of the cross-section and mass density of the beam, respectively; $m=\varrho A$ is the mass of the beam per unit length.

The strain energy U can be written as

$$U=\int_0^L\frac{1}{2}EI\kappa^2\,\mathrm{d}x=\int_0^L\frac{1}{2}EI\left(v''+\frac{1}{2}v''v'^2\right)^2\mathrm{d}x, \tag{4}$$

where E is the Young's modulus of the beam; I is the moment of inertia of the cross-section.

The variation of the virtual work of nonconservative forces δW can be written as

$$\delta W=\int_0^L[(p-q-c\dot{v})\delta v+\underline{\underline{M\delta\theta}}]\,\mathrm{d}x \tag{5}$$

where c is the damping coefficient; q is the subgrade reaction. Depending on the shear character of the soil medium, some different foundation models, i.e., the Winkler model, Pasternak model, Vlasov model and Kerr model (see Fig. 1c), are applied to simulate the action of soil medium. In this case, the subgrade reaction can be described by a general mathematical form: $q(x,t)=k_0v(x,t)-k_1v''(x,t)$ (see Appendix A). Furthermore, the last term ($M\delta\theta$) is the virtual work of the second-order moment, due to the component of the excitation and the subgrade reaction along the longitudinal direction of the beam (see the insert in Fig. 1). Therefore, the virtual work of the second-order moment can be written as

$$M\delta\theta=(M_p+M_q)\delta\theta=\left(\frac{ph\sin\theta}{2}+\frac{qh\sin\theta}{2}\right)\delta\theta. \tag{6}$$

3. Equation of motion

Substituting Eqs. (3)–(5) into Eq. (2), and using Eq. (6) and the general expression of the subgrade reaction, carrying out the conventional procedure of the calculus of variation, we can obtain the in-plane motion equation of the beam on the elastic foundation

$$m\ddot{v}+c\dot{v}+\underbrace{EIv''''-j\ddot{v}''+k_0v-k_1v''}_{\text{linear term}}=p-\underbrace{\frac{h}{2}[v'(k_0v-k_1v'')]'}_{\text{quadratic nonlinear term}}$$
$$\underbrace{-EI(v'v''^2+v'^2v''')'}_{\text{cubic nonlinear term}}-\underbrace{\frac{h}{2}\left(pv'+\frac{pv'^3}{2}\right)'}_{\text{parametric excitation term}}. \tag{7}$$

Overall, Eq. (7) reflects the contribution of the rotary inertia of the beam to the linear term ($-j\ddot{v}''$). Moreover, due to the second-order moment, additional quadratic nonlinear term and parametric excitation term are included in the present model. It should be pointed out that the present beam-foundation system includes the beam and part of the SSI, and the quadratic nonlinear term depends on the foundation parameter (k_0,k_1). Therefore, from the physical viewpoint, the second-order participation of the SSI may destroy the conservative character of the system.

To obtain more general conclusion, we introduce the following nondimensional parameters and variables

$$x^*|v^*=\frac{x|v}{L};\quad \lambda=\frac{h}{L};\quad J=\frac{j}{mL^2};\quad p^*=\frac{PL^3}{EI};\quad K_0=\frac{k_0L^4}{EI};$$

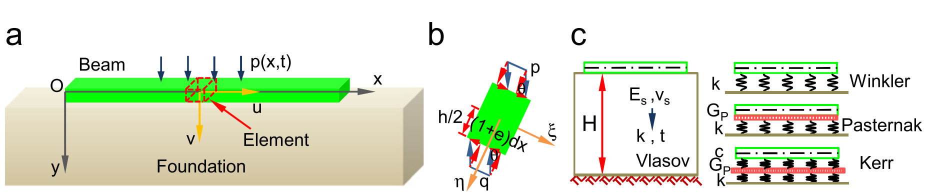

Fig. 1. The inextensional beam resting on the elastic foundation: (a) beam on elastic foundation, (b) beam element and (c) foundation models.