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Predictions of aneurysm formation in distensible tubes: Part A—Theoretical background to alternative approaches

Andrea Bucchi [a,*], Grant E. Hearn [b]

[a] School of Engineering, University of Portsmouth, United Kingdom

[b] Fluid Structure Interaction Research Group, Faculty of Engineering and the Environment, University of Southampton, United Kingdom

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ABSTRACT

Pressurised distensible tubes are subject to aneurysms. Aneurysm inception will take place at a location along the tube when a critical pressure, relative to tube wall thickness at that location, is reached. Parents will recognise the existence of critical pressure when endeavouring to inflate a party balloon. Another example of aneurysm is the thoracic aortic aneurysm corresponding to permanent dilation of the aorta in such proportions that it can be life threatening. Corrective procedures for aortic aneurysms involve the introduction of stiff materials to prevent aneurysm. Similarly in a proposed distensible tube based wave energy device aneurysm inception is partially controlled through the use of alternative longitudinal strips of inextensible material and appropriate rubber strips. Here we consider distensible tubes made of one material.

Having reviewed the aneurysm based literature some inconsistencies were observed between the material properties used in a non-linear finite element analysis and the material properties of the specimen used to provide experimental measurements for comparison. To appreciate the inconsistencies the authors decided to investigate aneurysm development using both non-linear finite element analyses and distinct alternative formulations and solution techniques. Rather than restrict strain-energy function to a subset of Neo-Hookean, Mooney–Rivlin and Ogden forms, the authors have implemented several alternative strain-energy models in parallel, also exploring for the first time the impact of using different combinations of uniaxial, equi-biaxial and pure shear experimental data for different rubbers.

This paper addresses the needs (necessary considerations, such as the Valanis–Landel hypothesis, Maxwell equal area rule and data selection criteria) for a realistic approach to modelling a distensible tube to provide predictions of critical pressure. In common with all other cited references a static analysis is used.

1. Introduction

Within the literature critical pressure is determined from static analysis with differing governing equations solved using alternative techniques. In the finite element approach strain-energy functions are utilised once the appropriate parameters have been determined from experimentally observed material behaviour. Of particular interest in this paper are the challenges and implications of using different experimental data sets and alternative strain-energy functions to determine the value of critical pressure. This is a problem shared in such diverse fields as bio-engineering, fluid-dynamics, mechanical and medical sciences.

The extent of research completed is too large for a single publication. Hence this paper presents the associated theory, with an indication of the derivation of the governing equations (omitted in all related papers found) and provides limited indicative qualitative comparative results. The companion paper presents direct numerical comparisons of predicted results for all different possible data set combinations and several alternative strain-energy functions. Predictions of the selected finite element package used by the authors are compared with an independently published generated finite element analysis.

This paper addresses the identification of a realistic approach to modelling a distensible tube to provide meaningful predictions of critical pressure. In common with all other cited references a static analysis is used.

Appreciation of balloon inflation is addressed by Osborne and Sutherland [1] and Müller and Struchtrup [2]. The investigation of aortic aneurysm is dealt with, in greater detail, by Vorp [3].

* Corresponding author. Tel.: +44 (0) 2392842401.
E-mail address: Andrea.Bucchi@port.ac.uk (A. Bucchi).

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Medical investigation of ruptured and intact aortic specimens indicates large diameter increases [4]. More complex tube construction using two distinct materials can lead to critical pressures beyond normal utilisation or occurrence. Medically this corresponds to the introduction of stents for reinforcement of arteries [5]. In engineering, the analysis of the more complex tube constructions used in wave energy devices ensures practical initial set up operational pressures consistent with wave environment. Whilst rubber is seen as a relatively low cost and low maintenance material it can experience different kinds of instability [6,7]. These examples, taken from different fields of interest, are sufficient to demonstrate the need to predict the likelihood and the prevention of an aneurysm. An appreciation of existing aneurysm research is summarised next.

1.1. Brief review of aneurysm research

Mallock [8] is credited with presenting the first paper addressing the formation of an aneurysm in a pressurised rubber tube. Mallock describes the birth of an aneurysm as 'one or more bulbous expansions' associated with the attainment of a critical radius and hence critical pressure.

Alexander [9] and Needleman [10] both investigated such instability during the inflation of spherical shaped rubber balloons. Haughton [11] has addressed perfect and imperfect spherical rubber membranes. Alexander [12] extended the instability studies to cylindrical membranes; Kyriakides and Chang [13,14] have provided theoretical and experimental comparison of the formation of aneurysms in cylindrical rubber tubes. Kyriakides and Chang observed experimentally that tube radius expanded uniformly along the tube length with increased pressure until critical pressure was attained. Thereafter radial growth was localised at the point of aneurysm initiation. As more fluid is injected into the tube the pressure is reduced and the aneurysm spreads longitudinally until the whole tube approximates a cylindrical shape. Further injection of fluid leads to increased pressure and radial growth until material failure occurs. The axial position of the aneurysm is influenced by manufacturing imperfections (non uniform wall thickness) or geometrical imperfections (radius variations along the tube) or even non homogeneous material properties.

According to Kyriakides and Chang [13,14] and Kanner and Horgan [15] critical pressure is the peak pressure in the pressure–volume curve. However, Fu et al. [16], indicate that the 'initiation pressure' associated with the initial bulge in the tube occurs within a small (mathematically perceived) neighbourhood of the critical pressure. It is defined as the pressure at which bifurcation occurs. This more precise definition befits theoretical bifurcation investigations (discussed briefly in Section 3.3.2) rather than experimental and engineering studies of aneurysm.

Kyriakides and Chang [14] solved stated governing equations using finite differences subject to fixed and rolled tube end condition. The slightly more detailed formulation of Guo [17] addresses a membrane of general axisymmetric shape. A simpler Guo [18] formulation restricts the initial shape of the membrane to be a uniform circular cylinder. Solutions [17,18] are generated using a 'shooting' technique (Section 14.1 of [19]) with both tube ends fixed. Yang and Feng [20] formally describe the 'shooting' technique as the application of the Cauchy–Kawalewski theorem when solving ordinary differential equations. That is, a two-point boundary value problem is treated as an initial value problem. The finite element approach can address both situations and with an appropriate choice of elements permits greater flexibility in tube end conditions and removal of assumed uniform material properties. Shi and Moita [21] developed an axisymmetric hyperelastic membrane and solved the non-linear static problem with the arc-length method [22]. Pamplona et al. [23] and Goncalves et al. [24] used the Riks algorithm with the ABAQUS® finite element software.

1.2. Comparison of numerical predictions and experimental measurements

Kyriakides and Chang [13,14] provide very comparable predictions and measurement of critical and propagation pressures, radial stretch ratio values at particular longitudinal positions and pressure variation with radial stretch ratio. Pamplona et al. [23] and Goncalves et al. [24] also report very good agreement between theoretical predictions and experimental measurements. However, closer analysis of [23,24] and the related PhD dissertation [25] (in Portuguese) suggest that material properties used in the predictions are quite distinct to those of the experimental study. Furthermore, the material stress–strain data is determined from a variant of the standard uniaxial tension test [26]. This limited data base and the apparent degree of agreement between prediction and measurement aroused our interest. In particular, Fig. 3 of [24] is generated using the strain-energy function parameters of Table 2 [24]. The close agreement of Neo-Hookean and Mooney–Rivlin fitted strain-energy functions with the experimental data is quite acceptable. However, the first-order Ogden model is quite distinct irrespective of adoption of Eqs. (15) or (16) of this paper. Furthermore, Table 2 [24] is different from Table 4.4 [25], although the corresponding force versus strain figures in [24,25] are identical. These different inconsistently attributed material properties provided the motivation to investigate the role of different selected material models upon critical pressure prediction in aneurysm formation.

1.3. Organisation of paper

Fundamental theoretical concepts and definitions regarding continuum mechanics for a hyper-elastic material are presented in Section 2.1. The principal steps to determine strain-energy function parameters from experimental data are presented with definitions of commonly used strain-energy functions attributed to Mooney–Rivlin, Ogden, Treloar (Neo-Hookean), Yeoh, Arruda–Boyce and Marlow in Section 2.2. Section 3 reviews static formulations of differing mathematical complexity for modelling and predicting aneurysm characteristics. Section 3.1 addresses a long cylindrical thin-walled tube, uniformly inflated, whereas a more complex analytic method for an axisymmetric membrane is treated in Section 3.2. The derivation of these two alternative semi-analytical methods are presented, together with summary algorithms, since the different predictions are studied to appreciate both the appropriateness of the finite element based modelling and to provide a general comparison; a task not previously reported in the literature. The authors' governing equations derivation within Section 3.2 is considered necessary given derivations have not been located in the extensive literature search undertaken. Hence they may be deemed useful to other researchers. The more general finite element method is summarised in Section 3.3 with justification of specific possible choices made regarding its application. Sample representative results for each technique are presented and discussed in Section 4. Finally closure concerning the different mathematical approaches is reported in Section 5. The original Treloar rubber data used [27] has been retrieved as explained in Appendix A. This data, rather than the Ogden fitted [28,29] Treloar rubber data, used by several authors [10,21,30], is utilised in our applications. A second set of tabulated data is due to Kawabata et al. [31]. The sensitivity of fitting different rubber models to different