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Size-dependent pull-in phenomena in nonlinear microbridges

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ABSTRACT

This paper investigates the deflection and static pull-in of microbridges based on the modified couple stress theory, a non-classic continuum theory able to predict the size effects for structures in micron and sub-micron scales. The beam is modeled using Euler–Bernoulli beam theory and the nonlinearities caused by mid-plane stretching have been considered. It is shown that modified couple stress theory predicts size dependent normalized deflection and pull-in voltage for microbeams while according to classical theory the normalized behavior of microbeams is independent of the size of the beam. According to results, when the thickness of the beam is in order of length scale of the beam material, the difference between the results given by modified couple stress theory and those predicted by classical theory is considerable.

1. Introduction

Microbeams are widely used in micro-electro-mechanical systems (MEMS), for example in micropumps, micromirrors, accelerometers and microswitches. Various actuation methods are used for the excitation of microbeams in MEMS. The Electrostatic actuation mechanism for the excitation has some merits such as fast response and simple drive electronics. A typical electro-statically actuated microbeam is indeed a straight microcantilever or microbridge having an initial distance from a rigid substrate, actuated by a transverse distributed electrical force caused by the input voltage applied between the microbeam and the substrate. As the voltage increases gradually, the deflection of the microbeam also increases. At a certain voltage, the microbeam may face the instability such that the deflection suddenly increases, and the microbeam contacts with the substrate through the point of maximum deflection (e.g. the middle point of the microbridge). The excitation voltage corresponding to the instability is called the static pull-in voltage. It is noted that if the rate of change of the excitation voltage is not negligible, the inertial effects of the beam are important. The voltage corresponding to instability in these dynamic conditions is called the dynamic pull-in voltage. In this work, the static pull-in voltage is investigated. The pull-in instability is one of the important phenomena that should be considered in the design, analysis and simulation of micro-electro-mechanical systems. This phenomenon is experimentally investigated by some researcher such as Nathanson et al. [1] and Taylor [2].

Many researchers have studied the static pull-in phenomenon. Here some of these works are reviewed. Mojahedi et al. [3] presented an analytical investigation for the static pull-in of microcantilevers and microbridges using the homotopy perturbation method. Bochobza-Degani and Nemirovaski [4] analyzed the pull-in of microcantilevers modeled as two-degree of freedom systems. Zhang and Zhao [5] studied the static pull-in of microbeams utilizing Taylor series expansion of the electrostatic force. Chaterjee and Pohit [6] presented a model for the analysis of static and dynamic pull-in instability of microcantilevers considering the nonlinearities caused by large deformation of the microbeam. Rong et al. [7] proposed a model to analytically calculate the pull-in voltage for multilayer clamped–clamped microbeams (microbridges) using an energy-based approach.

Beams used in MEMS have the thickness in the order of microns and sub-microns. Many experiments have been performed to observe the mechanical behavior of micro-structures. The experiments show that the normalized stiffness in small scales is size dependent. For example, some torsion experiments on thin copper wires with plastic deformation carried out by Fleck et al. [8] confirmed that considering only the strain hardening in modeling, gives results with less hardening than observed in the experiments. They observed that a decrease in wires diameter results in a considerable increase of the hardening. Also, Stolken and Evans [9] performed some tests on the bending of thin nickel microbeams. They reported a notable increase of plastic work hardening caused by the decrease of beam thickness in the micro-bending test of thin nickel beams with the thickness of 12.5, 25 and 50 μm. Moreover, the size-dependent behavior is detected in some kinds of polymers. For instance, Chong and Lam [10] conducted some bending tests on a number of microcantilevers

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made of epoxy in order to determine the significance of the size-dependency of the normalized bending rigidity. They observed that the normalized bending rigidity of the microbeams increases as the beam thickness decreases. They deduced that the normalized bending rigidity of the microbeams depends on the ratio of the beam thickness to the material length scale parameter. Also, in order to obtain the force–deflection curves, McFarland and Colton [11] used a nano-indenter to deflect the end of some microcantilevers made of polypropylene having a non-homogenous microstructure. They reported that the stiffness values obtained by the nano-indenter are observed at least four times greater than the values predicted by the classical elasticity theory. Wang et al. [12] showed that strain gradient theory can reduce the gap between experimental observation and theoretical results of natural frequency of electrostatically actuated microbeams. They proposed a length scale for polysilicon by fitting their results to the experimental finding on the vibration of electrostatically actuated microbeams. According to these experimental results, it is clear that the size-dependent behavior is an inherent property of a structure when its characteristic size, e.g. the thickness or the diameter of a beam, is comparable to the internal material length scale.

The classical continuum mechanics is not able to predict and explain the size-dependent behavior which occurs in micron and sub-micron scale structures. However, non-classical continuum theories such as the couple stress theory [13–16] are acceptably capable of interpreting the size-dependent behavior. In 1960s some researchers such as Koiter [13] and also Mindlin and Tiersten [14] introduced the couple stress elasticity theory as a non-classic theory capable of predicting the size effects. In this theory, beside the classical stress components acting on elements of materials, the couple stress components, as higher-order stresses, are also available. Specific constitutive equations express the couple stress components in terms of the gradient of the rotations of elements and some higher-order material constants. The higher-order material constants are usually related to the classical material constants by the length scale of the material.

Here a number of works which used the couple stress theory to investigate some structural phenomena is mentioned. The behavior of structures with cracks or grooves has been analyzed based on this theory by Ejike [15], and also Kishida and Sasaki [16]. Asghari et al. [17] employed the couple stress theory to investigate the size effects in Timoshenko microbeams.

A modified couple stress theory introduced by Yang et al. [18] by employing the equilibrium equation of moments of couples beside the classical equilibrium equations of forces and moments of forces. This newly established theory is employed successfully in order to predict the size-dependent static and vibration behavior of microbeams. As an example, utilizing the modified couple stress theory, Park and Gao [19] analyzed the static mechanical behavior of an Euler–Bernoulli beam and interpreted the outcomes of epoxy polymeric beam bending test. Also, Kong et al. [20] derived the governing equation, initial and boundary conditions of an Euler–Bernoulli beam based on the modified coupled stress theory using Hamilton's principle. In addition, based on the modified couple stress theory, the size-dependent natural frequencies of fluid-conveying microtubes, the size-dependent buckling behavior of micro-tubules and the size-dependent resonant frequencies and sensitivities of atomic force microscope (AFM) microcantilevers have been investigated by Wang [21], Fu and Zhang [22] and Kahrobaiyan et al. [23], respectively. Moreover, a new Timoshenko beam model based on the modified couple stress theory was formulated by Ma et al. [24]. They assessed the size-dependent static and free-vibration behavior of a simply-supported Timoshenko beam as a case study. Furthermore, employing the modified couple stress theory, the nonlinear Euler–Bernoulli and Timoshenko beam formulations have been developed by Xia et al. [25] and Asghari et al. [26], respectively. They showed that not only linear behavior, but also nonlinear static and dynamic behavior of microbeams are size-dependent.

In this paper, based on the modified couple stress theory, the static deflection and pull-in voltage of an electrostatically actuated microbridge is investigated considering the nonlinearities caused by mid-plane stretching and axial force. The governing equation is solved numerically and the convergence of the numerical method is shown. The obtained results are compared with the results corresponding to the classical theory. Results show that when the ratio of beam thickness to material internal length scales is not large, there is a notable gap between the results given by classical theory and modified couple stress theory.

2. Microbeam modeling

The modified couple stress theory is introduced by Yang et al. [18]. Based on this theory, the strain energy density for infinitesimal deformations is written as [20]

$$\overline{u}=\frac{1}{2}(\sigma_{ij}\varepsilon_{ij}+m_{ij}\chi_{ij})\quad (i,j=1,2,3), \tag{1}$$

where σ_{ij} and ε_{ij} denote components of the stress tensor and strain tensor, respectively, and

$$m_{ij}=2l^2\mu\chi_{ij}, \tag{2}$$

$$\chi_{ij}=\frac{1}{2}((\nabla\theta)_i+(\nabla\theta)_i^T), \tag{3}$$

in which m_{ij} and χ_{ij} stand for deviatoric part of the couple stress tensor, and the symmetric curvature tensor, respectively. Also l is the material length scale parameter. The components of the rotation vector θ_i are related to the components of the infinitesimal displacement vector field according to

$$\theta_i=\frac{1}{2}(curl(\mathbf{u}))_i. \tag{4}$$

Now consider a clamped–clamped microbeam of length L with initial stretching force N under electrostatic distributed loading as shown in Fig. 1. The cross section of the beam is assumed to be rectangular with width b and height (thickness) h.

For an Euler–Bernoulli beam, the displacement field is expressed as

$$u_1=u-z\frac{\partial w}{\partial x},\quad u_2=0,\quad u_3=w(x), \tag{5}$$

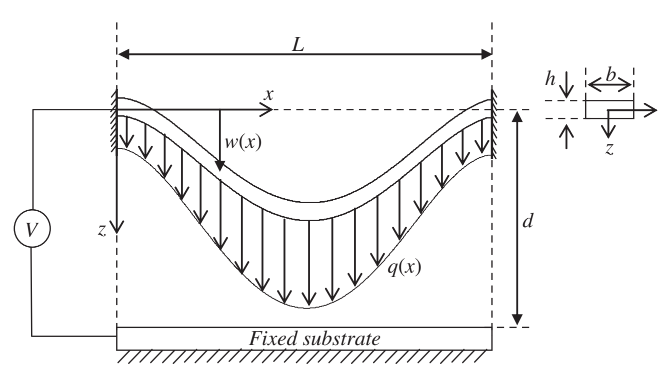

Fig. 1. An electrostatically actuated microbridge.