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The design of laminated glass under time-dependent loading

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ABSTRACT

Laminated glass is a layered sandwich structures composed of elastic glass plies bonded by viscoelastic polymeric interlayers, which produce the mechanical shear-coupling of the plies under flexural loads. Here, we analytically solve the time-dependent problem of a simply-supported three-layered sandwich-beam with linear-viscoelastic interlayer under a loading/unloading history, showing that its gross response is strongly affected by the rheological properties of the polymer, here modeled by Wiechert–Maxwell units. The results, confirmed by numerical simulations, are compared with those obtainable with an approximate solution, commonly used in the design practice, where the interlayer is modeled by an equivalent linear-elastic material, whose properties are calibrated according to temperature and characteristic duration of the applied loads. For this, practical design rules to account for superimposition of applied loads are proposed.

The qualitative properties of the two approaches are analytically discussed, evidencing those load-histories under which the approximate solution is, or is not, conservative for what stress and deflection evaluation is concerned.

1. Introduction

Laminated glass is a composite sandwich structure made of two or more glass plies bonded together by polymeric interlayers, with a process at high pressure and temperature in autoclave referred to as *lamination*. The interlayers are too soft and thin to present appreciable flexural stiffness, but nevertheless they can provided shear stress that constrains the relative sliding of the glass plies, thus increasing the stiffness and load bearing capacity of the composite package. In general, it is customary to identify [1] the two borderline cases of (i) interlayers with no shear-stiffness and free-sliding glass-plies as the *layered limit* and that of (ii) shear-rigid interlayers and perfectly bonded glass-plies as the *monolithic limit*.

In order to achieve a reliable and economical design, it is necessary to take into account the shear coupling provided by the interlayers, but its evaluation is complicated by the viscoelastic response of the polymer, that is highly time and temperature dependent. In the design practice it is common to consider approximate solutions, at various levels of accuracy. Geometric non-linearities are usually important because of the slenderness of the laminated panel [2,3], but can be neglected, at least as a first-order approximation, when the loads are mainly orthogonal to the panel surface and no in-plane forces are present. Furthermore, for what concerns the material behavior, the most used simplifying assumption is to consider that the polymer is a linear elastic material, whose elastic shear modulus depends on temperature and characteristic duration of the design actions. For ease of reference, such data are usually provided by manufactures under the form of tables. These are commonly obtained by performing creep tests under constant shear strain at various temperatures, and by measuring the shear stress as a function of time; it is then immediate to calculate the *secant stiffness* of the interlayer and the end of each characteristic time interval. Because of this, in the sequel this kind of approximate solutions will be referred to as the *secant stiffness solution* (SSS). As it will be widely discussed in the present paper, to assume the SSS approximation is equivalent to neglect the *memory effect* of the polymer, i.e. the dependence of the stress not only on the current strain but on the strain history. The use of SSS is particularly effective because there are several practical methods to readily calculate the response of laminated structures composed of linear elastic layers, such as those proposed by Newmark et al. [4], Bennison and Stelzer [5], Foraboschi [6], Galuppi and Royer-Carfagni [7] for the case of beams, and by Ašik [2], Foraboschi [8], Galuppi and Royer-Carfagni [9] and for the case of plates, to mention just a few.

In [10], the authors have considered the paradigmatic case of a simply-supported composite beam with viscolastic interlayer, under constant loading. The time-dependent problem has been solved analytically, in order to obtain the *full viscoelastic solution* (FVS), modeling the response of the polymer by various types of

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Prony's series for the Maxwell–Wierchert model [11]. Such model, that is the most general model of linear viscoelasticity, combines in parallel a series of Maxwell spring-dashpots units and, consequently, can take into account that relaxation does not occur at a single time-scale by at a number of different time-scales, each one associated with the corresponding Maxwell unit. The parameters that define the constitute properties may be found through creep or relaxation tests (see, for example [12,13]), or by measuring the response to cyclic oscillations [14,15]; in a just few cases, they are directly furnished by the manufactures. Temperature dependence may be taken into account using the Williams–Landel–Ferry model [16]. It should be mentioned, however, that the viscoelastic parameters vary considerably from polymer and polymer and, most of all, can be affected by the lamination process.

The solution of the time-dependent viscoelastic problem has evidenced noteworthy differences, at the qualitative level, between the FVS and SSS. In synthesis, the "memory effect" of viscoelasticity affects the gross response of the composite beams: the polymer *gradually* relaxes, letting the coupling shear stress *gradually* decrease with time, thus producing the progressive decay of the gross stiffness of the laminated beam. In the SSS, the stiffness of the polymer is the one it would exhibit if the strain had been kept constant for the whole time-history: this leads to an underestimation of the shear stress that can be transferred between the glass plies and, hence, of the overall stiffness and strength of the sandwich beam. Hence, the gross response of the laminated beam with viscoelastic interlayer appears stiffer than it would be when calculated according to the SSS approximation.

A general result of [10] has been that the SSS is always on the side of safety with respect to FVS when the load is kept constant with time. However, it is still an open question whether this conclusion holds under the most various time-dependent actions. Moreover, to our knowledge, in the technical literature it has never been clarified how to use the SSS when loads are applied and then removed, for example when the actions due to wind or snow are added to the dead load. In other words, what is the equivalent shear stiffness of the polymer that must be considered when superimposing the effects of loads with very diverse characteristic durations? This is a key point for civil structures always subjected to loads of various nature, but our personal experience is that, quite surprisingly, almost each designer has its own rule of thumb.

In this paper, we try to give an answer to the aforementioned questions elaborating the model presented in [10]. We analytically prove general properties of the FVS under the most various load-histories, independently of the particular Maxwell–Wierchert model assumed for the polymer. Moreover, the full-viscoelastic problem here proposed allows to rigorously consider the issue of load superposition. From this analysis, practical rules applicable to approximate methods of design, such as the SSS, are deduced. According to the proposed superposition method, the SSS under time-dependent loads is most of the times conservative with respect to the FVS, for example when the load history is monotone. However, it shown that there are particular cases, associated with very short-duration loads such as the impulsive actions due to impact, for which the SSS may underestimate the state of stress and deflection in the composite beams. In such cases, the full-viscoelastic analysis seems to be the only possible approach.

Although the application is here specifically addressed to laminated glass, nevertheless the proved results are of very general nature and apply to any kind of layered sandwich structures with viscoelastic interlayer. The applications are in civil engineering, as well as in automotive, aeronautics and shipbuilding, and may range from structural insulating panels, consisting in a layer of polymeric foam sandwiched between two layers of structural board, to steel beams supporting concrete slabs connected by ductile studs, to wood elements made of glued layers.

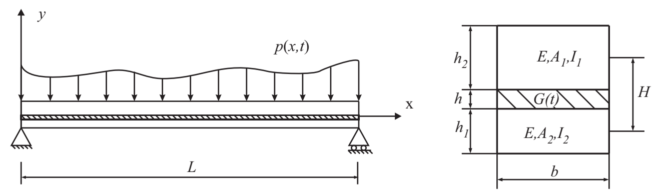

Fig. 1. Sandwich beam composed of two linear-elastic external layers, bonded by a viscoelastic interlayer.

2. Composite beams with viscoelastic interlayers

In order to discuss the effects of the interlayer viscosity on the response of laminated glass, it is convenient to refer to a specific model-problem, representative of the most usual conditions under which the composite is employed.

2.1. The model problem

As shown in Fig. 1, consider a simply-supported sandwich beam of length L, composed of two external linear elastic plies of thickness h_1 and h_2, bonded by a thin viscoleastic interlayer of thickness h. This is so thin to present no flexural stiffness, but stiff enough to transfer shear stresses between the external plies. The structure is loaded by the time-dependent force per unit length $p(x,t)$.

This model perfectly adapts to the case of laminated glass, where the external plies are made of glass, whereas the interlayer is a polymeric sheet. The two external glass layers present linear-elastic response, with the same Young's modulus E, whereas the interlayer is made of a viscoelastic polymer, with shear modulus $G(t)$, whose constitutive properties will be discussed in Section 2.2. The governing equations have already been derived in [7] for the particular case when the interlayer is linear elastic, but they can be specialized to the case of viscoelasticity with no particular difficulty.

With reference to Fig. 1, let us define

$$A_i = h_i b, \quad I_i = \frac{bh_i^3}{12}(i=1,2), \quad H = h + \frac{h_1+h_2}{2}, \quad A^* = \frac{A_1 A_2}{A_1+A_2},$$
$$I_{tot} = I_1 + I_2 + A^* H^2, \tag{2.1}$$

and observe that I_{tot} represents the moment of inertia of the full composite section, corresponding to the *monolithic* limit, i.e., no relative slippage occurs between glass plies. Under the hypothesis that strains are small and rotations moderate, the kinematics is completely described by the vertical displacement $v(x,t)$, the same for the three layers, and the horizontal displacements $u_1(x,t)$ and $u_2(x,t)$ of the centroid of the upper and lower glass layers, respectively. In the sequel, $(')$ will denote differentiation with respect to the variable x, whereas $(\dot{})$ will represent differentiation with respect to t. The transversal displacement $v(x,t)$ is assumed to be positive if in the same direction of increasing y, the transversal load $p(x,t) > 0$ if directed downwards, while the bending moment $M(x,t)$ is such that $M(x,t) > 0$ when $v''(x,t) > 0$.

As it is demonstrated by Galuppi and Royer-Carfagni [7], the shear strain in the interlayer, $\gamma(x,t)$, is constant through its thickness h and reads

$$\gamma(x,t) = \frac{1}{h}[u_1(x,t) - u_2(x,t) + v'(x,t)H]. \tag{2.2}$$