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The pressure distribution in nips of systems of flexible rubber-covered rollers

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ABSTRACT

In this paper, a simulation tool is presented to support the design process of systems of flexible, rubber-covered rollers. A mathematical model is developed to determine the axial pressure variations in the nips of these systems. In the model the indentations of the rubber layers are coupled to the deflections of the rollers due to bending. The model is generic in the sense that the number of rollers in the system is not fixed, and rollers may have different lengths and be located asymmetrically with respect to each other. The rollers bend due to linear conditions applied at arbitrary positions along the roller axes. The rollers may have a surface profile and the rubber layer thicknesses may vary. In each nip paper can be present, located at any position. By an efficient numerical algorithm that solves the model, the effect of bending on the nip pressure distribution is determined. Crowning, use of an additional support roller, and internal bearings are examples shown as means to achieve a uniform pressure distribution.

1. Introduction

In many industrial applications thin media transport between clamped rollers is used. We mention paper-making, sheet-metal and textile-drafting processes. In this paper we feel inspired by paper transport between rollers that are covered with rubber. The contact area between the rollers, the nip, is a crucial design aspect of the roller system. One of the design criteria is prevention of paper wrinkling due to differences in paper transport speed that are caused by variations of the pressure along the rollers. For an optimal design it is necessary to have a pressure distribution that is uniform.

The pressure distribution along the rollers strongly depends on the stiffness and thickness of the rubber layers covering these rollers. Hence, by tuning the rubber layer properties in combination with the clamping forces applied, a pressure distribution is obtained that meets the design criteria. In the design process a simulation tool is needed to determine the optimal configuration.

To reduce size and weight of systems the tendency is to use light and flexible rollers. The introduction of flexible rollers has a significant impact on the pressure distribution. Since the rollers bend easily the pressure distribution becomes non-uniform. In an optimal design the effect of roller bending is counteracted by alternatives that diminish the amount of bending. Thus, the simulation tool must be able to deal with any configuration, also one with more than two rollers. In this paper, we present a simulation tool to support the design process of systems of flexible, rubber-covered rollers.

To determine the pressure distribution in the contact problem of rubber-covered, cylindrical rollers, the assumption of plane-strain state, in which rollers are infinitely long and axial variations in pressure are ignored, is frequently used. If the contact width is small compared to the roller radii the contact problem is often solved by replacing the body locally by a strip that is loaded at one side over a finite region where displacements are prescribed. Johnson [1] and Kalker [2] show some half-plane problems in which the pressure distribution can be determined analytically. In most practical problems, however, the elastic layer is relatively thin and semi-analytical methods are used that describe the stresses in terms of Fourier integrals and match the boundary conditions in a discrete number of points. See, for instance, [3–7], where a strip indented by a rigid cylinder is considered. Nowell and Hills [8] also take slip behavior into account. Gupta and Wallowit [9] and Kalker [10] considered the indentation of a layered medium. The sliding contact within a coated elastic system has been investigated by many researchers, cf. [11–14]. Braat [15] and Soong and Li [16,17] also incorporate transport of paper between the elastic strip and indenting cylinder. The disadvantage of the plane-strain assumption is that axial variations in pressure cannot be analyzed, so that no information about the probability of paper wrinkling can be derived. Wang et al. [18] analyze the three-dimensional contact problem for an elastic layered half-space.

To analyze axial pressure variations Finite Element Methods are used to solve three-dimensional configurations. For instance,

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in [19] a three-dimensional rolling contact is evaluated for two rubber-covered cylindrical rollers that have a rigid core. Generally, a Finite Element Method requires large computation times and has difficulty to incorporate roller bending because of the small grid size needed.

Parish [20] takes the effect of bending into account in a two-dimensional contact between a rigid and a rubber-layered roller by using an empirical relationship for the amount of deflection. Riese [21], Singh and Paul [22], and Yoichiro and Yamaura [23] solve the Euler–Bernouilli beam equation to include bending. All authors consider two rollers, of equal length, that are placed precisely above each other and that bend under fixed boundary conditions given at the roller ends. Hinge and Maniatty [24] evaluate the pressure distribution for two skewed, cylindrical rollers by assembling various beam elements developed from a plane-strain state to incorporate bending. For each beam element a Finite Element Method is created; the presence of paper in the nip is ignored.

The model that we present in this paper consists of the Euler–Bernouilli beam equation for the bending of the cores of the rollers, while a non-linear algebraic equation obtained from the plane-strain state model of [15] for a layered medium describes the rubber indentations. It is used as a simulation tool in the design process of systems of paper transport. The model is generic in the sense that the number of rollers in the system is not fixed, and rollers may have different lengths and be located asymmetrically with respect to each other. In each nip a sheet of paper can be present, located at any position. The rollers bend due to linear conditions applied at arbitrary positions along the roller. Each roller can have a surface profile in longitudinal direction. The approach taken allows the inclusion of axial variations in rubber layer thicknesses, resulting from, for instance, crowning applied to the rollers or a rubber layer that has been worn out. In Section 2 we describe the mathematical model for the pressure distribution in all nips of the system. In Section 3 we present the numerical approach for solving the mathematical model. We demonstrate the strength of the approach in obtaining an efficient simulation tool. In Section 4 we give numerical results and compare the pressure distribution to the measured profiles of [24]. We evaluate crowning, use of an additional support roller and internal bearings as means to make the pressure distribution uniform. Finally, in Section 5 we discuss and present the conclusions of our work.

2. Mathematical model

We consider a configuration of N cylindrical rollers, cf. Fig. 1. All rollers have a core, usually made of metal, and are covered with a rubber layer. These cores, of radius R_n, can be either solid or hollow. The inner radius of a hollow core is denoted $R_{n,\mathrm{in}}$. Apart from the bending, the elastic deformation of the cores is assumed negligible compared to that of the rubber layers. Hence, during bending we assume the cores retain their cylindrical cross-section, which implies the cores are sufficiently long or the wall of a hollow core is sufficiently thick. For the typical lengths considered in Section 4 this means that the wall thickness $R_n - R_{n,\mathrm{in}}$ of a hollow core is more than 20% of the outer radius R_n.

The rollers have length l_n, $n = 1,\ldots,N$; their axes are placed in the (y,z)-plane, which we take as the global reference frame. The position of roller n ranges from $z = a_n$ to $z = b_n$, so that $l_n = b_n - a_n$. We define $a = \min\{a_n\}$, $b = \max\{b_n\}$, $L = b - a$. Between two consequent rollers a sheet of paper can be present; see Fig. 2. The rollers are pinched to each other by clamping forces.

Due to the clamping forces two consequent rollers form a region of contact, the nip. For a system of N rollers maximally

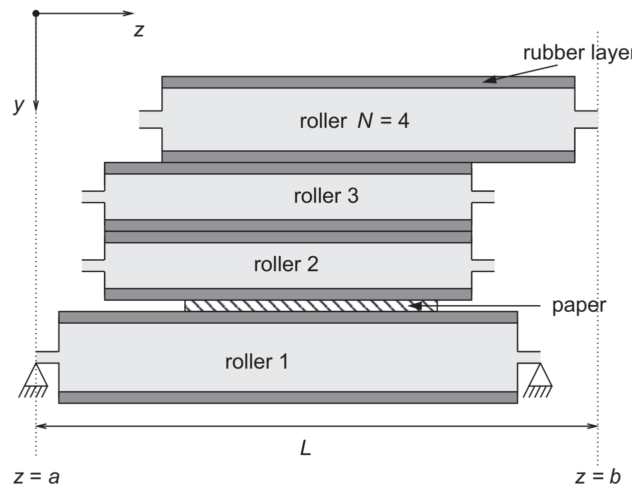

Fig. 1. A configuration of 4 rubber-covered rollers with a sheet of paper in the first nip in its undeformed state, as example for a general N-roller system.

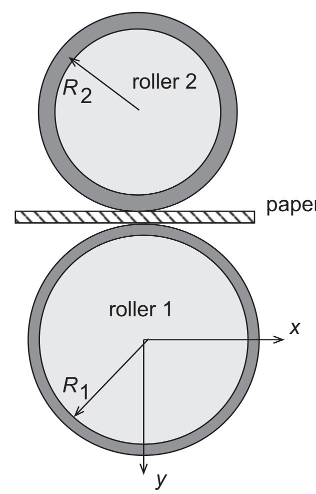

Fig. 2. Side-view of roller 1 and 2 of the configuration of Fig. 1, with a sheet of paper in the nip.

$N-1$ nips are formed. Paper present in a nip is assumed to be incompressible and to have no bending stiffness. As a result, paper acts as a spacer between rollers, and the pressure in a nip is the result of resistance against indentation of the rubber layers and bending of the cores of the rollers. Since the width of each contact region is small in comparison to the radius and length of the rollers we assume that there is line contact. The line pressure in the nip formed by two consequent rollers n and $n+1$ is denoted by p_n; the width of the nip after deformation is denoted by e_n; see Fig. 3.

2.1. Kinematic relation

The application of clamping forces results in a rigid body motion of roller n over a distance t_n in the positive y-direction in the global reference frame,

$$t_n(z) = \gamma_n + \omega_n(z - \bar{z}_n).$$

Here γ_n is the translation of the center of mass $\bar{z}_n = a_n + l_n/2$, and ω_n is the rotation of the roller around the x-axis. Besides the rigid body motion, roller n has an additional displacement δ_n due to its rubber indentation and a displacement u_n due to the bending of its core. By analyzing the situation before and after deformation we determine distance e_n between roller n and $n+1$ in the