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Homogenized elastic–viscoplastic behavior of plate-fin structures at high temperatures: Numerical analysis and macroscopic constitutive modeling

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ABSTRACT

In this study, homogenized elastic–viscoplastic behavior of an ultra-fine plate-fin structure fabricated for compact heat exchangers is investigated. First, the homogenized behavior is numerically analyzed using a fully implicit mathematical homogenization scheme of periodic elastic–inelastic solids. A power-law creep relation is assumed to represent the viscoplasticity of base metals at high temperatures. The plate-fin structure is thus shown to exhibit significant anisotropy as well as noticeable compressibility in both the elastic and viscoplastic ranges of the homogenized behavior. Second, a non-linear rate-dependent macroscopic constitutive model is developed using the quadratic yield function proposed for anisotropic compressible plasticity. The resulting constitutive model is shown to be successful for simulating the anisotropy, compressibility, and rate dependency in the homogenized behavior in multi-axial stress states.

1. Introduction

Plate-fin structures have been used as heat exchanger cores. Ultra-fine plate-fin structures are now under development for compact heat exchangers in high-temperature gas-cooled reactors (HTGRs) [1]. Fig. 1 shows a cross-section view of the plate-fin core trial-produced for HTGR [1]. To fabricate this core, 100 layers of plates and fins, with thicknesses of 0.5 and 0.2 mm, respectively, were stacked for brazing. More than 1000 layers of plates and fins need to be stacked to produce actual compact heat exchangers for HTGR. Full meshing of plates and fins in such heat exchangers necessarily results in extraordinarily large numbers of finite elements. It is therefore worthwhile to develop simplified methods for structural analysis of ultra-fine plate-fin structures.

Plate-fin structures are usually assumed to be periodic solids for evaluating their homogenized properties to develop simplified methods [2,3]. Homogenized properties of periodic solids can be exactly analyzed using the so-called mathematical homogenization method [4–6]. Elastic–viscoplastic homogenization based on this method was studied by Wu and Ohno [7] and Ohno et al. [8,9], and has been successfully applied to fiber-reinforced laminates and plain-woven laminates [10–12]. Recently, Asada and Ohno [13] formulated a fully implicit incremental homogenization scheme that is effective for periodic elastic–inelastic solids. This implicit scheme can be efficient for numerically evaluating the homogenized elastic–inelastic stress–strain relations of plate-fin structures.

The homogenized behavior evaluated as mentioned above needs to be represented using a macroscopic constitutive model to perform simplified structural analysis. Plate-fin structures generally exhibit significant anisotropy and compressibility in both the elastic and inelastic ranges, as will be shown in Section 3. Anisotropic compressible constitutive models of rate-independent plasticity were developed by Badiche et al. [14] and Xue and Hutchinson [15] using a quadratic yield function, which is an extension of Hill's orthotropic incompressible yield function [16] and is an orthotropic case of the quadratic strength function proposed for anisotropic materials by Tsai and Wu [17]. For isotropic solids, the quadratic yield function used by Badiche et al. [14] and Xue and Hutchinson [15] reduces to the isotropic compressible yield function considered by Green [18], Shima and Oyane [19], and Deshpande and Fleck [20]. These compressible yield functions have not been applied to the homogenized behavior of plate-fin structures yet.

At high temperatures, rate-dependent deformation such as viscoplasticity and creep may occur in plate-fin structures. Therefore, a non-linear rate-dependent anisotropic compressible constitutive model is necessary for representing the homogenized behavior of plate-fin structures at high temperatures.

In this study, homogenized high-temperature elastic–viscoplastic behavior of the plate-fin structure shown in Fig. 1 is investigated.

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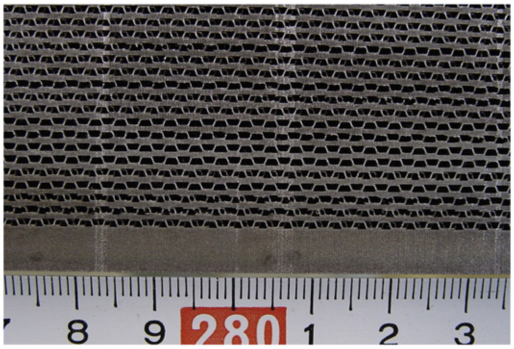

Fig. 1. Ultra-fine plate-fin core trial-produced for HTGR [1].

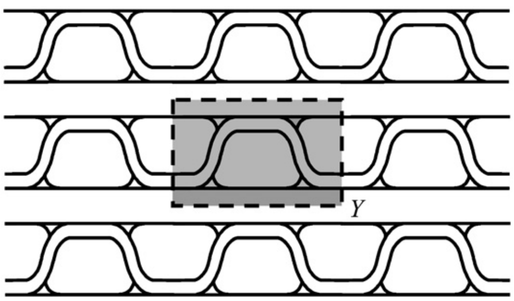

Fig. 2. Periodic plate-fin structure and unit cell Y assumed in this study.

First, the homogenized behavior is numerically analyzed using the fully implicit incremental homogenization scheme of periodic elastic–inelastic solids [13]. A power-law creep relation is assumed to express the viscoplasticity of base metals at high temperatures. Macroscopic anisotropy and compressibility of the plate-fin structure are thus discussed especially in the inelastic range. Second, a non-linear rate-dependent macroscopic constitutive model is developed using the quadratic yield function proposed for anisotropic compressible plasticity [14,15,17]. It is shown that the resulting constitutive model represents well the anisotropy, compressibility, and rate dependency in the homogenized elastic–viscoplastic behavior in multi-axial stress states.

2. Analyzed plate-fin structure

The plate-fin structure illustrated in Fig. 2 is considered in this study. It is assumed for simplicity that the fins have an in-phase laminar configuration and are straight in the direction of gas flow.[1] Then, since the plate-fin structure is periodic, the homogenized behavior can be evaluated by taking a unit cell Y. The shaded area in the figure is the unit cell Y analyzed in this study.

It is assumed that the plate-fin structure undergoes small deformation at high temperatures in the absence of body forces, and that strain $\boldsymbol{\varepsilon}$ is additively decomposed into elastic strain $\boldsymbol{\varepsilon}^{\mathrm{e}}$ and viscoplastic strain $\boldsymbol{\varepsilon}^{\mathrm{vp}}$:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{\mathrm{e}} + \boldsymbol{\varepsilon}^{\mathrm{vp}}. \tag{1}$$

[1] In fact, offset fins are used in the ultra-fine plate-fin structure shown in Fig. 1. However, the offset may not significantly influence the homogenized behavior, and is hence ignored in this study.

It is further assumed that all metallic parts, including brazed parts in the plate-fin structure, have the same material properties and obey elastic and viscoplastic constitutive relations:

$$\boldsymbol{\varepsilon}^{\mathrm{e}} = \frac{1+\nu}{E}\boldsymbol{\sigma} - \frac{\nu}{E}(\mathrm{tr}\,\boldsymbol{\sigma})\mathbf{I}, \tag{2}$$

$$\dot{\boldsymbol{\varepsilon}}^{\mathrm{vp}} = \frac{3}{2}\dot{\varepsilon}_0\left(\frac{\sigma_{\mathrm{eq}}}{\sigma_0}\right)^{n-1}\frac{\mathbf{s}}{\sigma_0}, \tag{3}$$

where E and ν are elastic constants, $\boldsymbol{\sigma}$ denotes stress, tr indicates the trace, $\mathbf{I}$ signifies the second rank unit tensor, the superposed dot indicates differentiation with respect to time t, $\dot{\varepsilon}_0$, σ_0, and n are the material parameters of viscoplasticity, $\mathbf{s}$ denotes the deviatoric part of $\boldsymbol{\sigma}$, and σ_{eq} expresses the Mises equivalent stress defined as

$$\sigma_{\mathrm{eq}} = \left(\frac{3}{2}\mathrm{tr}\,\mathbf{s}^2\right)^{1/2}. \tag{4}$$

The plates and fins are brazed using a filler metal; as a result, residual stress may occur after cooling from a brazing temperature. Such residual stress is locally important [21], but is ignored for a simple discussion of homogenized behavior in this study.

3. Homogenized behavior of the plate-fin structure

3.1. Method of homogenization analysis

The homogenized elastic–viscoplastic behavior of the plate-fin structure described in the preceding section was numerically analyzed using the fully implicit incremental homogenization scheme developed by Asada and Ohno [13]. This scheme is based on the mathematical homogenization method, in which Y-periodicity is imposed on the perturbed component $\tilde{\mathbf{u}}$ of displacement $\mathbf{u}$ at the boundary of a unit cell Y. Using the scheme, we can efficiently compute not only homogenized stress–strain relations but also distributions of micro-stress $\boldsymbol{\sigma}$ and micro-strain $\boldsymbol{\varepsilon}$ in Y.

From here on, homogenized stress and strain will be, respectively, referred to as macro-stress $\boldsymbol{\Sigma}$ and macro-strain $\mathbf{E}$, which are related to $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ by

$$\boldsymbol{\Sigma} = \frac{1}{|Y|}\int_Y \boldsymbol{\sigma}\,\mathrm{d}Y, \tag{5}$$

$$\mathbf{E} = \frac{1}{|Y|}\int_Y \boldsymbol{\varepsilon}\,\mathrm{d}Y, \tag{6}$$

where $|Y|$ denotes the volume of Y. Moreover, for convenience, $\boldsymbol{\Sigma}$ and $\mathbf{E}$ will be vectorially expressed as

$$\boldsymbol{\Sigma} = \{\Sigma_{xx} \quad \Sigma_{yy} \quad \Sigma_{zz} \quad \Sigma_{xy} \quad \Sigma_{yz} \quad \Sigma_{zx}\}^{\mathrm{T}}, \tag{7}$$

$$\mathbf{E} = \{E_{xx} \quad E_{yy} \quad E_{zz} \quad \Gamma_{xy} \quad \Gamma_{yz} \quad \Gamma_{zx}\}^{\mathrm{T}}, \tag{8}$$

where the superscript T indicates the transpose, and Γ_{ij} $(=2E_{ij}, i \neq j)$ denotes engineering shear macro-strain.

3.2. Finite-element model of the unit cell

To perform homogenization analysis of the plate-fin structure, the unit cell Y illustrated in Fig. 2 was divided into three-dimensional eight-node non-conforming finite elements (Fig. 3). Cartesian coordinates x, y, and z were set for Y, as shown in Fig. 3. The thickness of Y in the z-direction was arbitrary because no dependence of $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ on z was presumed; as a result, only one element was taken in the z-direction. Hastelloy X at 900 °C was