International Journal of Mechanical Sciences 52 (2010) 706–715

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Post-failure behaviour of impulsively loaded circular plates

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ARTICLE INFO

Article history:
Received 11 February 2009
Received in revised form
27 October 2009
Accepted 17 November 2009
Available online 22 November 2009

Keywords:
Circular plate
Impulsive loading
Shear failure
Residual kinetic energy
Energy absorption

ABSTRACT

The dynamic plastic response of a simply supported circular plate is analysed. Emphasis is given to the plate behaviour after it has broken free from the supports due to a local material failure. The theoretical rigid plastic analysis predicts various features of the response such as the time to failure, residual kinetic energy and the critical velocity at failure. The residual kinetic energy of the plate could be significant enough to cause secondary impact damage. It is shown that the shape of the plate changes after breaking free from the supports, which is important for forensic investigations. The solution for various cases were proven to be exact in the context of the upper and lower bounds theorems of the theory of plasticity.

1. Introduction

The transverse impact of an object on a plate might lead to a local material failure at the impact point, eventually causing penetration depending on various factors such as the material strength and the impact speed. On the other hand, a blast load which acts on a plate may cause it to fail around the supports, as it has been shown in Refs. [1,2]. This type of failure can be explained by noting that blast loads might produce significant transverse shear forces, particularly at the supports or other hard points. This has been shown analytically using a rigid plastic idealisation in Ref. [3], where it has been observed that the transverse shear force tends to infinity for impulsive loadings.

In such a scenario, a structure may fail at the supports and travel unimpeded through space. This behaviour has been analysed in Refs. [4,5] for simply supported and clamped beams, respectively. These analyses did reveal new features of beam dynamics besides relating a simple failure criterion to a continuum damage mechanics model. For instance, after the beam breaks free from its supports, a plastic hinge travels in the beam towards its centre, causing further plastic deformation as the beam travels through space. This behaviour is of interest for forensic studies, which examine the damage in structural systems after a major accident.

The visco-plastic response of free–free beams [6], or beams colliding one against the other [7–10] has received attention in the literature, but no similar studies appear to have been published on plates and shells. Thus, in the present study, a similar analysis to that presented in Refs. [4,5] for beams is reported here but for the more complex case of a simply supported circular plate. The next section of this article gives the equilibrium equations for circular plates. Sections 3–5 present the theoretical rigid plastic solution to the posed problem for various plate strengths, with Section 6 discussing the solutions and closing the article.

2. Equilibrium equations

Fig. 1 shows an infinitesimal element of a circular plate of thickness H together with the generalised stresses considered in the present analysis. Equilibrium of this element requires that [3]

$$\frac{\partial}{\partial r} rM_r - M_\theta - rQ_r = 0 \tag{1}$$

and

$$\frac{\partial}{\partial r} rQ_r + rp = \mu r\ddot{w}, \tag{2}$$

where r and θ are the cylindrical coordinates, M_r and M_θ are the unit radial and circumferential bending moments, Q_r the unit transverse shear force, p the distributed load, μ the plate mass per unit area and w the plate midplane transverse displacement, with the double dots representing the second derivative of time, t.

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doi:10.1016/j.ijmecsci.2009.11.014

Nomenclature

H	plate thickness
I_1	defined by Eq. (94)
k	material constant
K	kinetic energy
K_b	total bending energy
K_f	energy in bending after severance
K_i	initial kinetic energy
K_m	kinetic energy when hinge reaches the plate centre
K_p	total plastic energy
K_r	residual kinetic energy
K_s	energy absorbed by plastic shear hinge
M_r	radial bending moment
M_θ	circumferential bending moment
M_0	bending moment for plastic collapse of cross-section
Q_r	transverse shear force
Q_0	transverse shear force for plastic collapse of cross-section
r	radial coordinate
R	plate radius
T	total time duration
t	time
t_f	final time
t_{fa}	time to failure
t_m	time when the moving hinge reaches the plate centre
t_r	time when rigid body motion is initiated
V_0	impulsive velocity
V_{cr}	threshold impulsive (critical) velocity
V_r	rigid body velocity
w, W	transverse plate displacement
$_{c_f}, _{s_f}$	index to variables for the post-failure motion
$_{c_m}, _{s_m}$	index to variables during the propagating hinge phase of motion
$_{c_r}, _{s_r}$	index to variables when rigid body motion starts
W_f	final plate displacement
W_{fa}	plate transverse displacement at failure
W_s	transverse shear displacement at the plate support
W_c	transverse shear displacement at the plate centre
$W_{\bar{c}}$	transverse centre displacement at the plate support at failure
$W_{\bar{s}}$	transverse shear displacement at the plate support at failure
$\dot{w}_s$	transverse shear velocity at the plate support
β	defined by Eq. (71)
Δt	defined by Eq. (107)
Γ	defined by Eq. (64)
κ	plate curvature
λ	defined by Eq. (34)
μ	plate mass per unit area
ν	defined by Eq. (8)
Ω	defined by Eq. (35)
ρ	material mass per unit volume
σ_0	material flow stress
θ	circumferential coordinate
ξ	plastic hinge position
ξ_0	initial plastic hinge position
$(\dot{\ })$	time derivative of ( )
$(\ddot{\ })$	second time derivative of ( )

The kinematics of this infinitesimal element implies that the curvatures in the r and θ directions are

$$\kappa_r = -\frac{\partial^2 w}{\partial r^2}, \quad (3)$$

$$\kappa_\theta = -\frac{1}{r}\frac{\partial w}{\partial r}, \quad (4)$$

respectively, where membrane strains are disregarded since neither in-plane loads or finite displacements are considered in the present analysis.

The yield condition shown in Fig. 2 relates the radial and circumferential bending moments and the transverse shear force. It will be shown later that to adopt a transverse velocity profile for the plate which is either constant or varies linearly with the radius leads to an exact solution. Therefore,

$$\dot{\kappa}_r = 0 \quad \text{and} \quad \dot{\kappa}_\theta \geq 0, \quad (5)$$

which implies that $M_\theta = M_0$ is always satisfied and that M_r remains to be determined from the analysis, with M_0 being the plastic bending strength moment of the plate cross-section of unit width given by

$$M_0 = \frac{\sigma_0 H^2}{4}. \quad (6)$$

Moreover, the transverse shear force, Q, may assume different values depending on the boundary condition and time.

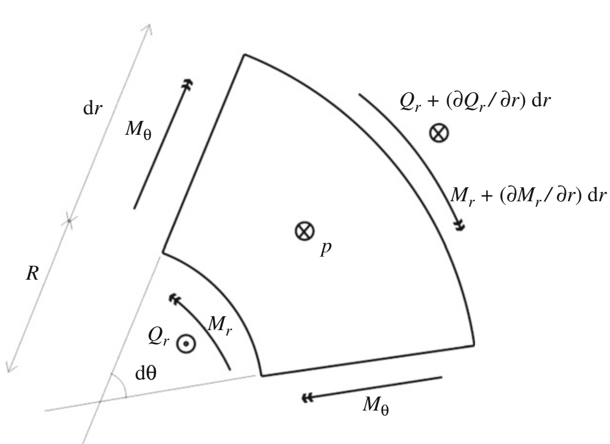

Fig. 1. Circular plate infinitesimal element.

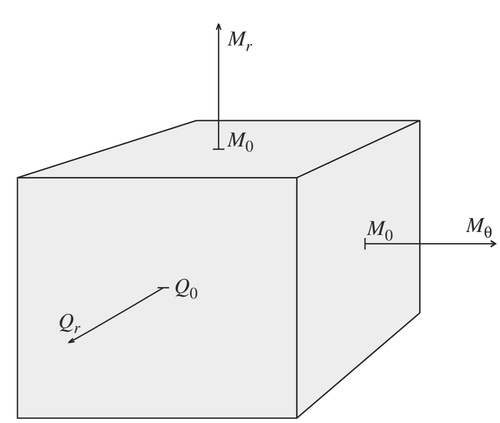

Fig. 2. Yield surface used in the analysis.