International Journal of Mechanical Sciences 53 (2011) 606–616

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Bending analysis of a laminated composite patch considering the free-edge effect using a stress-based equivalent single-layer composite model

Jaehun Lee [a], Maenghyo Cho [a], Heung Soo Kim [b,*]

[a] School of Mechanical and Aerospace Engineering, Seoul National University, Gwanak 599, Gwanak-ro, Gwanak-gu, Seoul 151-742, Republic of Korea
[b] Department of Mechanical, Robotics and Energy Engineering, Dongguk University-Seoul, 26 Pil-dong 3-ga, Jung-gu, Seoul 100-715, Republic of Korea

ARTICLE INFO

Article history:
Received 18 November 2010
Received in revised form
29 April 2011
Accepted 18 May 2011
Available online 27 May 2011

Keywords:
Composite patch
Free-edge effect
Interlaminar stress
Flexure model
Stress function
Complementary virtual work principle

ABSTRACT

The interlaminar stresses of a laminated composite patch, which is made up of reinforcing fibers (carbon/graphite) and epoxy matrix are analyzed using a stress-based equivalent single-layer model under a bending load. The composite patch is frequently used as reinforcement for a metallic adherend of mechanical/aerospace structures (i.e., aluminum alloy, etc) by attaching the film- or paste-type adhesive (i.e., epoxy, BMI, etc). To calculate the adhesive stresses transferred from the substrate, an interlayer model is introduced. The adhesive stresses are obtained by solving the equilibrium equations. The stress fields of the patch are determined by assuming certain stress functions. To satisfy the equilibrium state of the patch, the stress functions are divided into homogeneous and particular parts. The adhesive stresses act as prescribed stress boundary conditions of the laminated composite patch. The stress functions are substituted into a complementary virtual work principle, and from this, two coupled ordinary differential equations are obtained. General eigenvalue problems are derived to solve the coupled governing equations. To demonstrate the validity and efficiency of the proposed method, cross-ply, angle-ply and quasi-isotropic laminated composite patches are studied. From the observations made, the authors found that the stress function-based approach is suitable for solving the stress prescribed boundary value problem with accuracy and efficiency compared to a displacement-based approach such as the finite element method. The proposed method can be used as an efficient tool in the initial design stage of structural components when it is necessary to consider the free-edge effect.

1. Introduction

Composite materials have been used in various mechanical and aerospace structures due to their favorable characteristics such as high strength-to-weight and stiffness-to-weight ratios. In particular, laminated composite patches are often used to repair and reinforce metallic host structures as well as being an inherent part of the main structures. Patching techniques, which use laminated composites for repair, play an important role in delaying the initiation and growth of cracks in damaged structures [1]. In the practical usage of these composite patches, there are two different methods for repairing the host structure: bonded patching and bolted patching. Bonded composite patches are more popular than bolted patch since the bolted patch is affected by the risk of further damage occurring due to stress concentration around bolt holes, whereas this is obviously not a problem for bonded patch. In addition, bonded patches are both convenient in the repair of and efficient in strengthening damaged structures. Due to these advantages, the analysis and design of bonded patches has been the subject of numerous papers.

Volkersen [2], Goland and Reissner [3] proposed classical models of bonded patches. Closed-form solutions were obtained from the classical models, and these solutions gave a reference for estimating the stress distribution in adhesives and adherents [4–6]. More recently, considering bi-axial loading conditions, Mathias et al. [7] extended the unidirectional problem to a bi-dimensional rectangular one, which considers two-dimensional stress fields. Moreover, an experimental investigation based on a full-field measurement method [8] was conducted with accurate results. However, there has never been a study on stress concentration within the laminated composite patch—or the so-called free-edge effect. The free-edge effect occurs due to the discontinuity of material properties in the composite laminates. The major phenomenon of the free-edge effect is a local, strong stress concentration at the interface of each lamina in order to reach the stress equilibrium condition when external loads are applied. Thus all composite structures, which contain free-edges, including the laminated composite patch, show interlaminar stress concentrations.

Even these days the free-edge effect is a serious problem, it causes delaminations and damage to simple cross-plys as well as

* Corresponding author. Tel.: +82 2 2260 8577; fax: +82 2 2260 9397.
E-mail address: heungsoo@dongguk.edu (H.S. Kim).

0020-7403/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijmecsci.2011.05.007

arbitrarily stacked composites. In order to investigate the mechanical near the free-edge, numerous approximate methods have been derived because of the difficulties that occur during the procedure of calculating the exact elasticity solutions. After the pioneering work of Pipes and Pagano [9], displacement-based layer-wise theories (DBLWT) [10–12] and displacement-based single-layer theories (DBST) [13,14] have been proposed to study the free-edge problem. Displacement-based approaches are convenient to derive finite element formulations. However, although finite element analysis has become more and more popular simple and accurate stress-based methods, which guarantee the equilibrium state near the free-edge, are still desirable to predict interlaminar stresses in the initial design stage.

After Spilker and Chou [15] proposed a stress-based method, Kassapoglou and Lagace [16] developed a force-balance method based on stress-based layer-wise theory (SBLWT). However, the results of this method do not satisfy the point-wise traction-free boundary condition at the free-edge. Yin [17,18] used piecewise polynomial approximations, and his solution satisfies the free-edge boundary conditions in a point-wise sense. Layer-wise theories, however, are a little complicated compared to the stress-based equivalent single-layer theory (SBEST). SBEST is commonly based on the stress functions, which satisfy the equilibrium condition. The major advantage of the SBEST compared to the stress-based layer-wise theory (SBLWT) is a computational efficiency since the degrees of freedom in the SBEST are lower than those in the SBLWT. Nevertheless, the accuracy can be maintained by introducing series type stress functions [19] and by applying an iteration process [20,21].

Flanagan [19] proposed an efficient SBEST approach based on a series expansion of beam eigenmode shapes. Flanagan's approach predicted accurate in-plane interlaminar stresses, but oscillations appeared in the through-the-thickness direction. To overcome these oscillations of interlaminar stresses in the through-the-thickness direction, Cho et al. [20,21] proposed the extended Kantorovich method. In this method the converged stresses were obtained not only in the axial extension case but also in bending, twisting, and thermal loading cases. Kim et al. used this methodology for strength analyses [22] and for examining the internal ply-drop problem [23]. In addition, layup optimizations with a genetic algorithm (GA) were conducted under various loading conditions [24–26].

Recently, Kim et al. [27] modified Flanagan's SBEST [19] and analyzed the free-edge effect of a laminated composite patch under uni-axial extension with accurate results. In the present study, the authors extended the above method to the analysis of the free-edge effect on a laminated composite patch under a bending load. Unlike the regular free-edge problems, the analysis of a composite patch requires additional stress functions, which account for the load transfer mechanism from the host structure to the composite patch. Thus, in this paper, new particular stress functions are introduced, which is in a similar line of thought as of the previous work of Kim et al. [27] but is not the same. In this method the loading condition is different from the uni-axial extension in Ref. [27]. Moreover, the prescribed stress boundary conditions on the bottom of the composite patch are calculated using an interlayer model of Alfredsson et al. [28] in order to describe the bending load applied to the metallic substrate. The stress distributions of the composite patch obtained by the proposed stress function-based methods are compared to those obtained by the commercial finite element package, NASTRAN.

2. Statement of the problem

The composite patched structure consists of three layers, a metallic substrate, a thin adhesive layer and a laminated composite patch. The schematic of the composite patched structure is presented in Fig. 1. In the present study, the substrate is in a purely intact state, and the composite patch is perfectly bonded to the substrate. The bending load is applied to the substrate, and the transverse normal and shear loads are transferred through the adhesive layer.

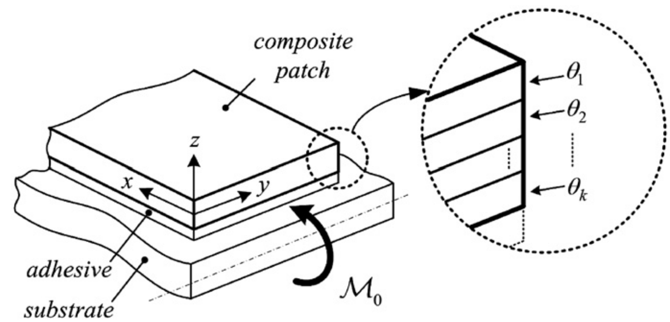

Fig. 1. Geometry and loading condition of the composite patch.

The calculation of the solution consists of two steps. The first step is calculating the adhesive stresses, which are transverse normal and transverse shear stresses that are transferred from the host structure to the bottom of the composite patch. In this step, the adhesive stresses can be obtained using the interlayer model in Ref. [28], which assumes the composite patch as an equivalent single layer. This method solves the equilibrium equations of the three layered asymmetric structure which contains the interlayer. To obtain the equilibrium equation, the Euler–Bernoulli beam type model in which the displacement fields are divided into symmetric and asymmetric parts is introduced. The interlayer, which acts as an adhesive, has a compressive degree of freedom. The details of this process will be discussed in Section 3.

The second step is calculating the interlaminar stresses of the composite patch using the obtained adhesive stresses from step 1. In the second step, a modified SBEST is used. The stress field, which satisfies the equilibrium condition, is obtained by the Lekhnitskii stress function [29] and complementary virtual work principle. If the composite laminate is under an axial loading (extension, bending, twisting, etc.), the original SBEST is effective. However, when the stresses are transferred from the bottom of the laminates, one can express the load transfer mechanism by adding particular terms to the stress function. The method of adding these particular parts to the stress function was already applied and verified for the analysis of tapered laminates with internal ply-drops in Ref. [23]. The method will be discussed in more detail in Section 4.

3. Adhesive stresses of bonded composite patch

In this section, we followed the method described by Alfredsson et al. [28] and modified the boundary condition of the configuration given in that paper to adopt an interlayer model of our composite patch problem. Among the various flexure models, the model of Alfredsson et al. [28] is the most suitable for our patched structure with perfect bonding. To obtain the adhesive stresses simply and efficiently, we assumed a semi-infinite problem that only considers x-directional variation.

3.1. Governing equations for asymmetric interlayer model

The governing equations are obtained from the free body diagram (see Fig. 2) of the small elements of layered structures