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Modal characteristics of symmetrically laminated composite plates flexibly restrained at different locations

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ABSTRACT

A method for determining modal characteristics (natural frequencies and mode shapes) of symmetrically laminated composite plates restrained by elastic supports at different locations in the interior and on the edges of the plates is presented. The classical lamination theory together with an appropriate set of characteristic functions are used in the Rayleigh–Ritz method to formulate the eigenvalue problem for determining the modal characteristics of the flexibly supported laminated composite plates. Sweep-sine vibration testing of several laminated composite plates flexibly restrained at different locations on the plates is performed to measure their natural frequencies. The close agreement between the experimental and theoretical natural frequencies of the plates has verified the accuracy of the proposed method. The effects of elastic restraint locations on the modal characteristics of flexibly supported laminated composite plates with different lamination arrangements and aspect ratios are studied using the present method. The usefulness of the results obtained for predicting sound radiation behavior of flexibly supported laminated composite plates is discussed.

1. Introduction

Because of their many advantageous properties, composite materials are used to make structural parts in different industries. For instance, laminated composite plates are used to fabricate aircraft structures in the aerospace industry, vehicle parts in the automotive industry, and flat-panel speakers in the audio industry. In general, laminated composite plates used to fabricate these structures/parts are joined together via flexible connectors or restrained by other structural components, which act as flexible supports to the plates. For aircraft and vehicle structures, the determination of the actual dynamic behavior of the flexibly supported plates in the structures is an important task if unwanted vibrations or noise radiated from the structures are to be suppressed. For laminated composite flat-panel speakers, it is also important to determine the actual dynamic properties of the flexibly supported laminated composite sound radiating plates of the speakers if the generation of high-quality sound is desired. In general, the motion of a flexibly supported plate consists of two parts, namely vertical rigid body motion and flexural vibration. The vertical rigid body motion contributes mainly to the sound radiation of the plate in the low-frequency range while the flexural vibration contributes to that in the mid- to high-frequency range. In designing a flat-panel speaker, since sound quality in the audible frequency range depends on the motion of the sound radiating plate, the determination of accurate modal characteristics of such plates has become an important topic of research.

In the past several decades, many researchers have studied the free vibration of plate structures and proposed different techniques to determine their natural frequencies [1–10]. For instance, Ashton [1] used the Ritz method to study the natural frequencies and modes of free anisotropic square plates. In his study, the effects of anisotropy on the natural frequencies and mode shapes were investigated. Leissa [2] attempted to present comprehensive and accurate analytical results for the free vibration of rectangular plates. Hung et al. [3] presented an eigenvalue formulation for the free vibration analysis of symmetrically laminated rectangular plates with elastic edge restraints. In their study, the first 10 natural frequencies were determined. Ding [4] used a set of static beam functions in the Ritz method to study the free vibration of thin isotropic rectangular plates with elastic edge restraints. In general, most of the papers on free vibration analysis of plates have focused on the determination of their modal characteristics for regular boundary conditions, e.g., simply supported edges or flexible supports around the whole periphery. It is not uncommon that for specific applications, plates may be flexibly supported at particular locations inside or along their boundaries. Regarding the free vibration of such plates, it seems that few, if any, investigations have been reported. If deeper understanding of the modal characteristics of plates with different kinds of elastic supports is to be achieved, research investigation must be initiated.

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In this paper, the Rayleigh–Ritz method is used for the free vibration analysis of laminated composite plates supported by elastic restraints at different locations in the interior and along the edges of the plates. Experiments are performed to measure the natural frequencies of several laminated composite plates supported by elastic restraints at different locations in order to verify the accuracy of the proposed method. A number of examples are presented to quantify the effects of the positions of the restraints on the modal characteristics of laminated composite plates with different lamination arrangements and aspect ratios.

2. Plate vibration analysis

A thin rectangular symmetrically laminated composite plate of length a_o, width b_o, and constant thickness h composed of n layer groups is supported by elastic strips of width b_S and depth h_S at portions of the periphery of the plate and excited by a circular moving coil-type shaker in the interior of the plate as shown in Fig. 1(a). Herein, the rigid frame is used to position the edge elastic strips and shaker. As shown in Fig. 1(b), the damper together with the voice coil of the shaker works as an interior circular elastic restraint of radius r_c to support the plate in which the fiber angle of the kth layer group is θ_k. In the plate vibration analysis, the length and width of the plate are a and b, respectively, the edge elastic restraints are modeled as distributed rotational and translational springs, and the interior elastic restraint is modeled as a ring-type spring system of radius r_c composed of distributed translational springs as shown in Fig. 2(a). Herein, the x- and y-coordinates of the x–y–z Cartesian coordinate system lie in the mid-plane of the plate. Fig. 2(b) shows the locations of the elastic restraints in the x–y plane. For instance, the center of the interior ring-type spring system is located at (x_c, y_c), y_{p-1} and y_p are the coordinates of the end points of the pth edge elastic support on the edge at $x=0$, and x_{q-1} and x_q are the coordinates of the end points of the qth edge elastic support on the edge at $y=0$. In the present model, the edge springs are located at one half of the widths of the edge elastic restraints and the length and width of the plate are $a = a_o - b_S$ and $b = b_o - b_S$, respectively. For the laminated plate, if the deflection of the plate is $w(x, y)$, based on the classical lamination theory [11], the strain energy of the plate, U_p, can be expressed as

$$U_P = \frac{1}{2}\int_0^a \int_0^b \left[D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + 2D_{12}\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right) + D_{22}\left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 4D_{66}\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2 + 4D_{16}\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial x\,\partial y}\right) + 4D_{26}\left(\frac{\partial^2 w}{\partial y^2}\right)\left(\frac{\partial^2 w}{\partial x\,\partial y}\right) \right] dy\,dx \tag{1}$$

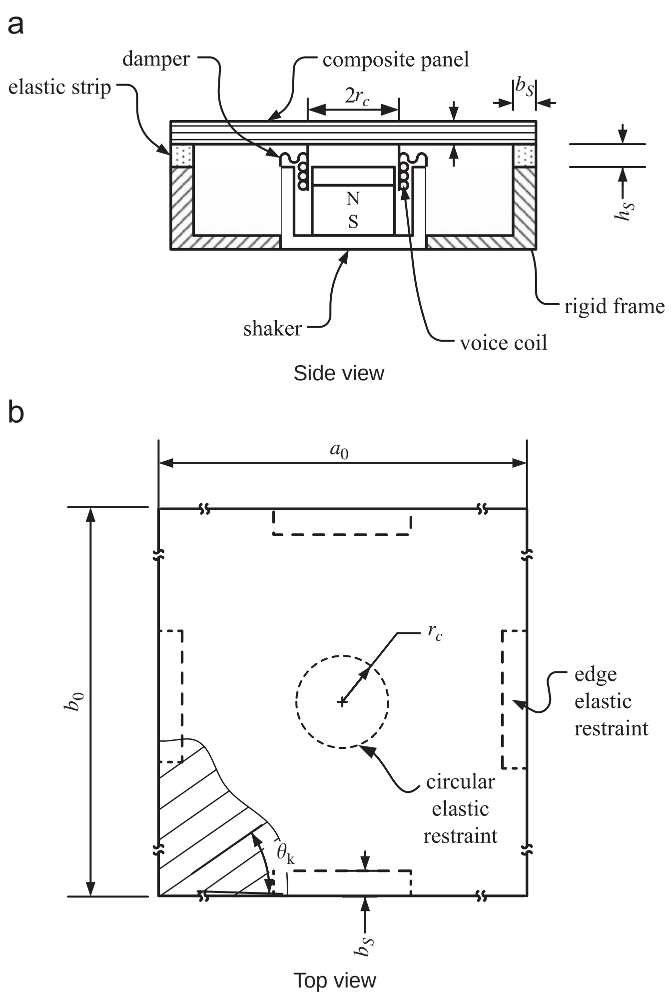

Fig. 1. Flexibly supported laminated composite plate excited by a moving coil-type shaker.

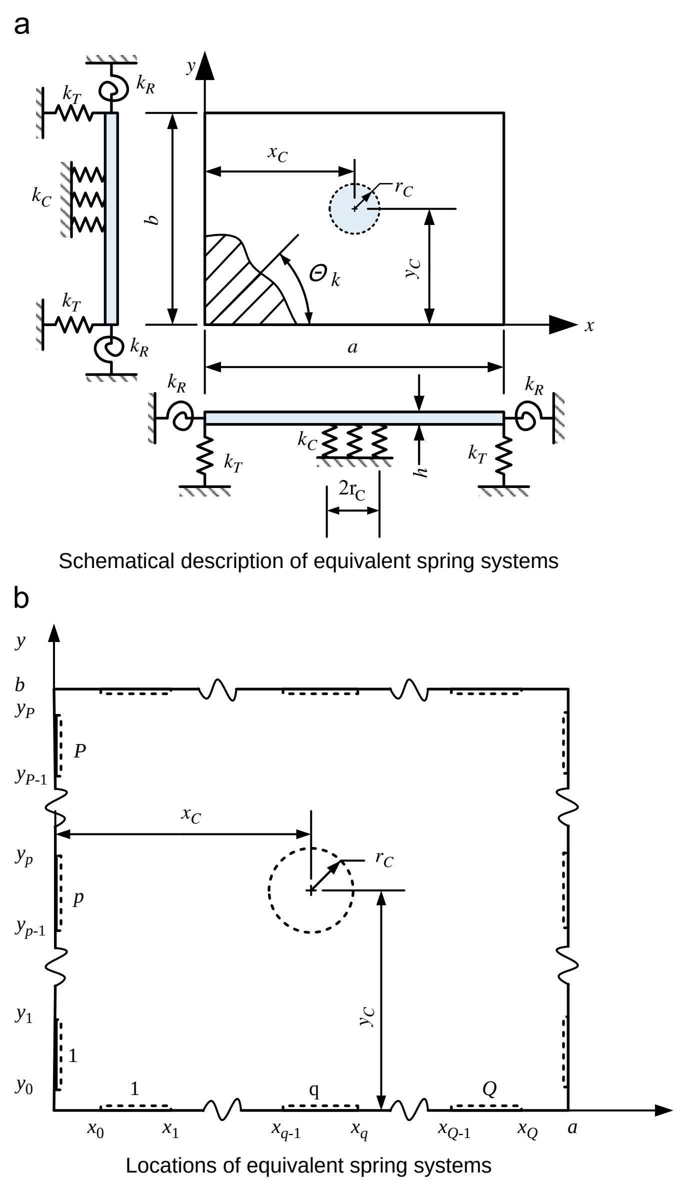

Fig. 2. Mathematical model of elastically supported plate.