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Transition failure stress in a chain of brittle elastic beads under impact

Dongxin Liu, Luming Shen *, Francois Guillard, Itai Einav

School of Civil Engineering, The University of Sydney, NSW 2006, Australia

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ABSTRACT

High strain rate experiments of chains of glass beads under impact loading demonstrate that first failure develops in either the first or second glass bead, depending on the impact velocity and associated stress propagation. Motivated by this notion of 'transition failure stress' we present a series of three-dimensional dynamic simulations of a 10 brittle elastic beads chain under impact of a stiffer elastic bar using the material point method (MPM). The numerical simulation results show that, as observed in the experiments, failure would first form in either the first or second closest brittle elastic bead to the impacting bar, depending on the parametric ratio between an impact-induced maximum tensile stress to fracture strength. Further non-dimensional analysis by varying materials and system parameters in the simulations suggests that this transition failure stress exists universally in brittle materials under impact loading.

1. Introduction

Stress waves in granular materials have received extensive attention [1–6]. In *brittle* granular materials, stress waves can create further implications related to fabric collapse and grain crushing [7–9]. They are pivotal in geoscience, for example in studying earthquakes fault gouges and meteoritic impacts [10,11], and in many industries, including addressing mining and mineral processes, petroleum production, and in pharmaceutics [12–14]. A key challenge is that the propagation of stress waves is controlled by the discrete nature of these materials and the dynamic behaviour of the individual grains. Most studies have dealt with stress waves where permanent irreversible effects such as damage and failure of the material may be disregarded [1]. However, in brittle granular materials the effect of permanent deformations on the propagation of stress waves should be considered [7,8]. Grain crushing may occur after the passing of strong stress waves, with fragments 'rattling' in the free voids [15]; this enhances alterations of porosity, stiffness, and permeability, all being crucial properties for predicting hydromechanical response of geo-environmental modifications. For example, Valdes et al. [16] revealed new forms of fragmentation waves in systems with intrinsically porous brittle grains, in the form of compaction bands propagating either periodically or intermittently. This distinction in the modes of propagation has recently been explained by Guillard et al. [17] through a new heuristic lattice spring model along with newly discovered dynamic compaction patterns in brittle porous media.

Due to the effects of the interactions among contacting grains under impact loading and the resulting damage, fracture and fragmentation of the individual grains, the propagation of stress wave in multi-grain system can be very complicated. Hence, studying the response of a chain of brittle spheres under impact loading can be a simple and effective way to begin tackling this complicated problem. Job et al. [18] conducted research on the law of solitary wave propagating in the linear chain of beads using experiments and numerical simulations. They proved that the characteristics of solitary waves could be influenced by the mechanical properties of the reflection boundary. Pal et al. [19] investigated the effects of plasticity in wave propagation in a chain of elasto-plastic granular system using the finite element method. The simulations revealed that energy dissipation could lead to the formation and merging of wave trains, which have characteristics that are very different from those of elastic chains. More recently, Wang and Nesterenko [20] studied, experimentally and numerically, the attenuation of short and strongly nonlinear stress waves in dissipative chains of alternatively arranged cylinders and spheres. They demonstrated that pulses in systems with a smaller cylinder to sphere mass ratios attenuate faster.

Motivated by the abovementioned problems, this paper explores the following question: in a chain of brittle beads, which bead is likely to fail first under an impact-induced stress wave? This question has been motivated by our recent impact tests on chains of glass beads. Specifically, these tests show that it is often the second closest bead to an impacting bar that fractures first among all the loaded beads. The question here is whether this phenomenon merely

* Corresponding author. School of Civil Engineering, The University of Sydney, NSW 2006, Australia. Tel.: +61 2 93512126; Fax: +61 2 93513343.
E-mail address: luming.shen@sydney.edu.au (L. Shen).

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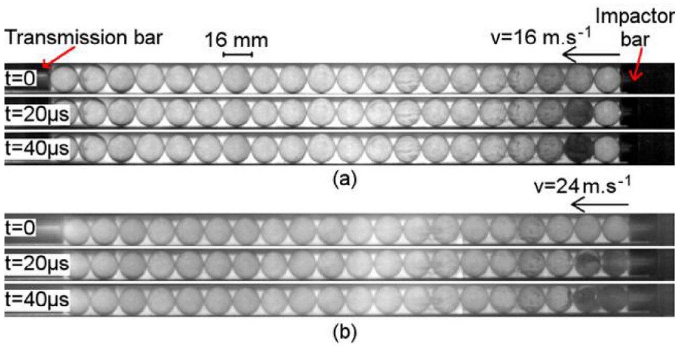

Fig. 1. Side view of the experimental impact tests of chains of glass beads in split Hopkinson bar device, using high-speed camera at 50,000 fps, for impact velocity of **(a)** 16 m/s and **(b)** 24 m/s. Darkening of a bead indicates complete splintering.

depends on material heterogeneities, or whether it may also be dictated by parameters such as impact velocity, duration of pulse or friction between granular particles, etc.

To better understand our previous experiments, we perform three-dimensional material point method (MPM) simulations to investigate the behaviour of a chain of 10 brittle elastic beads under the impact of a stiffer bar. Following the setup of the experiments, in the first numerical simulations, the mechanical properties of steel have been assigned to the bars ($E_s = 210$ GPa, $\rho_s = 7800$ kg/m^3), while those of glass have been assigned to the beads ($E_g = 70$ GPa, $\rho_g = 2500$ kg/m^3). The MPM is an extension of the particle-in-cell (PIC) method in computational fluid dynamics [21] to computational solid dynamics [22–24]. The motivation of the development was to study those problems with history-dependent internal state variables, such as contact/impact, penetration/perforation and fragmentation without invoking master/slave nodes and global remeshing. The MPM takes advantages of both Eulerian and Lagrangian methods. The mesh distortion problem in Lagrangian method can also be avoided through mapping to a mesh that can be controlled by the user [25]. Moreover, the MPM enables to naturally identify contacts between grains and to address the mesh distortion issues common in mesh-based methods due to the large deformation of particles [26].

The purpose of this study is to understand how stress wave propagation may induce damage in chains of brittle elastic grains under impact and to investigate if the friction coefficient, the initial impact velocity and the length of the impacting bar will affect the failure patterns in the chain using the 3D MPM simulations. More importantly, we aim to explain how the position of the first failure of a bead in the chain is influenced by the abovementioned factors as a function of the material strength and impact velocity.

2. Methods and procedures

2.1. Impact experiments

The numerical study performed has been motivated by experimental observations of the breaking of glass beads in a split Hopkinson bar device. In these experiments, a horizontal row of 20 glass beads kept in a channel are impacted at high velocity by a steel bar. The side view of the row of beads is recorded using a high-speed camera at 50,000 fps and the beads undertaking fracture get darker due to the increased diffusion of light where the glass is damaged. Fig. 1a shows that at low impact velocity, the second closest bead to the impacting bar is being severely damaged first, whereas the first closest remains almost intact. When increasing the impact velocity (Fig. 1b), we observe concurrent severe failure in the first and second closest beads to the impacting bar. These experiments suggest that there may be a transition between the low velocity case, where the first bead remains intact or slightly damaged, to a higher velocity case, where it completely breaks. Repetitions of these experiments provide similar outcomes, and therefore the reason for this phenomenon must mostly be related to deterministic material properties. The focus of this paper is therefore to explore this transition computationally by varying deterministically both system and model parameters.

Table 1
Different simulation parameters.

Impact velocity (m/s)	Length of impacting bar (mm)	Friction coefficient
2.5	200	0.4
1.5 5 10	200	0.4
2.5	100 300 400	0.4
2.5	200	0 0.2 0.6

2.2. Numerical simulation

To understand the transition in the breaking behaviour of a chain of glass beads upon impact, we perform numerical simulations using the MPM available in Uintah software [27]. The proposed problems and the simulation results are visualised using VisIt [28]. Fig. 2 shows the simulated problem where a stiff elastic bar impacts a chain of brittle elastic beads. As shown in the figure, the brittle elastic beads of diameter $D = 14$ mm form a horizontal chain contacting the left bar with 14 mm diameter and 200 mm length as a base. On the right end of the beads chain, another stiff elastic bar that shares the same length and diameter as the left bar impacts on the first elastic bead. In the simulation, the elastic bars and beads are axially aligned. The initial velocity of the impacting bar is 2.5 m/s. The friction coefficient of the beads in this reference simulation is 0.4.

In order to find whether the initial impact velocity, the length of impacting bar and the friction coefficient will affect the dynamic responses of the brittle elastic beads to impact loading, different simulations are carried out, as shown in Table 1. The first row of Table 1 illustrates the initial conditions of the reference simulation. In the other simulations listed in the second to fourth rows, the impact velocity, the length of impacting bar and the friction coefficient respectively are varied while keeping the other parameters fixed. However, we observe that the dynamics is only slightly affected by the friction coefficient. Indeed, this is confirmed when using different friction coefficients, as outlined in the fourth row of Table 1 while keeping the other parameters the same as those in the

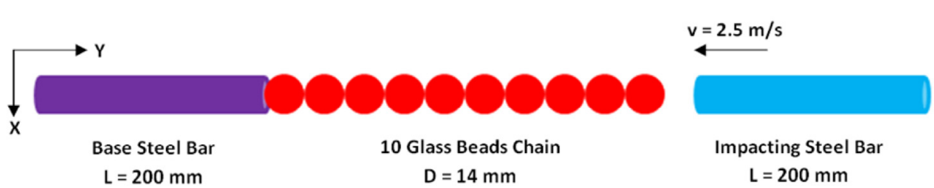

Fig. 2. Schematic of the simulated impact problem.