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Limit analysis of strain-hardening viscoplastic cylinders under internal pressure by using the velocity control: Analytical and numerical investigation

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ABSTRACT

The paper aims to assess plastic limit loads of thick-walled hollow cylinders of strain-hardening viscoplastic materials under internal pressure. Particularly, the problem concerned features in the interaction between strengthening and weakening behavior during the deformation process. Therefore, the relating onset of instability and the stability condition also deserve to be further investigated. Analytical and finite-element limit analysis efforts are both made for complete and comparative investigation. By the concept of sequential limit analysis, the plastic limit loads were acquired by solving a sequence of limit analysis problems via computational optimization techniques. Applying the velocity control as a computational strategy to simulate the action of pressure, the paper investigates analytically and numerically the plastic limit load, the onset of instability and the stability condition of plane-strain circular cylinders. Especially, analytical solutions of the onset of instability were solved explicitly by the fixed point iteration. Validation of the present analytical and finite-element efforts was made completely with good agreement between the analytical solutions and the numerical results.
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1. Introduction

Plastic limit load of cylinders is useful information requested frequently for an optimal structural design. As it is well known, limit analysis is a direct method to capture the asymptotic behavior of an elastic–plastic material by the lower bound or the upper bound theorem. Moreover, finite-element limit analysis [e.g. 1–15] further enhance the accuracy of limit analysis and broaden its applicability to more complex problems in engineering applications by taking advantage of techniques of finite-element methods [16] and mathematical programming [17]. On the other hand, if we consider structures made of strain-hardening viscoplastic materials, it is appropriate to evaluate the load-bearing capacity by limit analysis sequentially to illustrate the interesting interaction between strengthening and weakening behavior during the deformation process. By sequential limit analysis, it is to conduct a sequence of limit analysis problems with updating local yield criteria in addition to the configuration of the deforming structures. In each step and therefore the whole deforming process, rigorous upper bound or lower bound solutions are acquired sequentially to approach the real limit solutions. Accordingly, efforts [18–30] have illustrated extensively that sequential limit analysis is an accurate and efficient tool for the large deformation analysis.

In this paper, we consider the limit analysis problem of a plane-strain cylinder under internal pressure. The thick-walled cylinder considered is made of strain-hardening viscoplastic materials. Thus, it is not only a typical limit analysis problem aimed to seek the plastic limit loads sequentially, but it is also an interesting problem involving the interaction of strengthening and weakening behavior reflecting the properties of the strain-hardening and the strain-rate sensitivity during the deformation process. The strengthening behavior is due to from the material hardening properties. And the weakening phenomenon is corresponding to the strain-rate sensitivity and the widening deformation of a pressurized cylinder. Thus, it also deserves to pay attention to the onset of instability and the stability condition of the plastic limit load. Note that, the onset of instability concerned is about the plastic instability marked by the limit load maximum while dealing with thick-walled cylinders [31,32]. Namely, the strengthening due to material hardening is exceeded by the weakening resulting from the strain-rate sensitivity and the widening deformation. On the other hand, it is well known in the elastic–plastic numerical analysis that the action of internal pressure can be simulated either by using the stress (or load) control or by using the velocity (or displacement) control. Identified by the simulation method of the action of pressure load, two different normalization conditions were adopted in the computational procedures of finite-element limit analysis

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Nomenclature

a_0	initial interior radius
a	current interior radius
$\dot{a}$	current velocity of the interior radius
b_0	initial exterior radius
b	current exterior radius
$\{C\}$	coefficient matrix relating to the incompressibility constraint
D	problem domain
∂D_s	static boundary
∂D_k	kinematic boundary
E_U	convergence tolerance
G	a constant relating to the velocity control
h	hardening exponent
$[K]$	assembled stiffness matrix
$[K_{e1}]$	element stiffness matrix
$[K_{e2}]$	element stiffness matrix
m	strain-rate sensitivity
$\vec{n}$	unit outward normal vector of a boundary
N_e	number of elements used to discretize the domain
$p(\{U\})$	discretized inner product of the incompressibility constraint
P_i	internal pressure
q	load factor
$q(\sigma)$	lower bound functional
$\bar{q}(\vec{u})$	upper bound functional
q^*	exact limit load
$\tilde{q}(\{U\})$	finite-element discretized upper bound functional
$\tilde{q}(\{U^*\}_j)$	finite-element discretized upper bound functional calculated in the ith iteration
R	yield strength ratio
S	length of the innermost edge
$\vec{t}$	scalable distribution of a traction vector
t	transposition superscript
Δt	step size
Δt_i	step size in the ith iteration
$\{U\}$	nodal-point velocity vector
$\{U\}_0$	arbitrarily starting value of nodal-point velocity vector
$\{U\}_{j+1}$	unknown nodal-point velocity vector in the $(i+1)$th iteration
$\{U^*\}_j$	nodal-point velocity vector calculated in the ith iteration
$\vec{u}$	velocity field
$\vec{u}_s$	velocity field prescribed at the static boundary
$\Vert\cdot\Vert_2$	Euclidean norm
$\Vert\sigma\Vert_\vee$	von Mises primal norm on stress tensor
$\Vert\dot{\varepsilon}\Vert_{-\vee}$	von Mises dual norm on strain-rate tensor
σ	stress tensor
σ_r	stress component in the radial direction
σ_Y	yield strength
$(\sigma_Y)^n_{j+1}$	yield strength updated in the $(i+1)$th iteration of the nth step
σ_0	initial yield strength
σ_∞	saturation value of yield strength
$\bar{\sigma}$	equivalent stress
$\bar{\varepsilon}$	equivalent strain
$\bar{\varepsilon}^1$	equivalent strain for the first step
$\bar{\varepsilon}^n$	equivalent strain for the nth step
$\dot{\varepsilon}$	strain-rate tensor
$\dot{\bar{\varepsilon}}$	equivalent strain rate
$\dot{\bar{\varepsilon}}_0$	reference strain rate
$\dot{\bar{\varepsilon}}^n_{j+1}$	equivalent strain rate updated in the $(i+1)$th iteration of the nth step
δ	small real number
∇	vector differential operator
β	penalty parameter

[20–30]. In the stress control approach, the normalization condition is based on the simulation of the action of pressure load by imposing a uniform stress (pressure) field [20–22,25, 29,30]. In the velocity control approach, the normalization condition is obtained by simulating the action of pressure load with a uniform velocity field [23–24,26–28]. Particularly, in the finite-element limit analysis of circular hollow cylinders under internal pressure [26–28], we adopted the velocity (or displacement) control approach with the innermost edge expanded uniformly at a constant speed in the radial direction. It is noted that all the previous work [8,20–30] were conducted numerically by using a combined smoothing and successive approximation (CSSA) algorithm presented by Yang [33]. Particularly, the author and his co-worker extended the CSSA algorithm [33] with rigorous convergence analysis and validation to sequential limit analysis of viscoplasticity problems [26], or/and involving materials with nonlinear isotropic hardening [27–29].

The paper is aimed to analytically and numerically investigate the interesting interaction of strengthening and weakening behavior of pressurized cylinders made of strain-hardening viscoplastic materials. By the concept of sequential limit analysis, the plastic limit loads are acquired by solving a sequence of limit analysis problems via computational optimization techniques based on the CSSA algorithm [33]. Meanwhile, the velocity control is employed as a computational strategy to simulate the action of pressure. The resulting onset of instability and the stability condition corresponding to the velocity control are firstly investigated analytically in the paper to fully reveal the strengthening and weakening interaction. It is also noted that the Norton-Hoff viscoplastic model is utilized in the previous work [26,28] to consider the strain-rate sensitivity as utilized in regularized limit analysis [34]. On the other hand, the current work involving the strain-rate sensitivity is based on the rigid-plastic model with the updating yield strength step-wisely.

2. Problem formulation

We consider a plane-strain viscoplastic problem of the von Mises-type material with nonlinear isotropic hardening. It is noted that such problems feature in involving hardening material properties and weakening behavior corresponding to the strain-rate sensitivity in addition to widening deformation. The purpose is to seek the plastic limit load of a pressurized thick-walled hollow cylinder. Naturally, the problem statement leads to the lower bound formulation. By employing duality theorems [e.g. 8,13], we can establish the corresponding upper bound formulation from the lower bound formulation and further theoretically equates the greatest lower bound to the least upper bound. Therefore, we can approach the real limit solution by maximizing the lower bound or by minimizing the upper bound.

2.1. Problem statement (lower bound formulation)

We consider a general plane-strain problem with the domain D consisting of the static boundary ∂D_s and the kinematic boundary

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