International Journal of Mechanical Sciences 50 (2008) 1650–1657

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Nonlinear cylindrical bending analysis of shear deformable functionally graded plates under different loadings using analytical methods

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ARTICLE INFO

Article history:
Received 9 July 2007
Received in revised form
10 July 2008
Accepted 22 August 2008
Available online 20 November 2008

Keywords:
Functionally graded plate
Shear deformation
Cylindrical bending
Nonlinear analysis

ABSTRACT

An exact solution is presented for the nonlinear cylindrical bending and postbuckling of shear deformable functionally graded plates in this paper. A simple power law function and the Mori–Tanaka scheme are used to model the through-the-thickness continuous gradual variation of the material properties. The von Karman nonlinear strains are used and then the nonlinear equilibrium equations and the relevant boundary conditions are obtained using Hamilton's principle. The Navier equations are reduced to a linear ordinary differential equation for transverse deflection with nonlinear boundary conditions, which can be solved by exact methods. Finally, by solving some numeral examples for simply supported plates, the effects of volume fraction index and length-to-thickness ratio are studied. It is shown that there is no bifurcation point for simply supported functionally graded plates under compression. The behavior of near-boundary areas predicted by the shear deformation theory and the classical theory is remarkably different.

1. Introduction

In recent decades, non-homogenous materials such as composites have found extensive use in engineering applications mostly because of their many advantages such as specific strength or thermal properties. In some applications such as thermal protection systems, due to the low melting point of metals, the use of non-metallic coatings such as ceramics on a metallic substrate is enforced. These structures have some disadvantages because of the different physical and mechanical properties of the two different layers and the discontinuity at the interface. In recent years, a new group of materials called functionally graded materials (FGMs) has been developed to overcome these disadvantages. FGMs are made of two or more different materials with gradually variable material constituents and therefore there is no sharp variation in mechanical and physical properties of these materials.

Praveen and Reddy [1] analyzed the nonlinear static and dynamic response of FG ceramic–metal plates in a steady temperature field using the Finite Element Method (FEM) based on First-order Shear Deformation Theory (FSDT). Cheng and Batra [2] used the method of asymptotic expansion to study the 3D thermoelastic deformations of an FG elliptic plate. Static response of FG plates was studied using the generalized shear deformation theory by Zenkour [3]. He analyzed the stress and displacement response of the plates under uniform loading and showed that the deflection of the plates that correspond to properties intermediate to that of the metal and ceramic necessarily lies in between that of the ceramic and the metal. This behavior was found to be true irrespective of boundary conditions (BCs). A simply supported FG rectangular plate of medium thickness subjected to transverse loading was investigated by Chi and Chung [4,5]. They showed that the effect of changing Poisson's ratio on the mechanical behavior of the FG plates is very small. The large deflection of FG plates under pressure load was studied using the von Karman strains by GhannadPour and Alinia [6]. Through-the-thickness stress distribution in the aluminum and alumina plates is linear whereas for the FG plates the behavior is nonlinear and is governed by the variation of the properties in the thickness direction. Using a similar method employed by Sun and Chin [7,8], Navazi et al. [9] performed nonlinear cylindrical bending analysis of FG plates based on the Classical Plate Theory (CPT). They showed that the linear plate theory is inadequate for analysis of FG plates even in the small deflection range. Also, the behavior of the FG plates subjected to in-plane loading does not lie between those of pure ceramic and metal plates. Thermal postbuckling and vibration analyses of FG plates were studied by Park and Kim [10] using the nonlinear FEM based on the FSDPT. They showed that the behaviors of the FG plates do not necessarily lie between those of the isotropic plates with the ceramic and the metal. Although they did not underscore this finding in their conclusions, the most

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doi:10.1016/j.ijmecsci.2008.08.010

noteworthy point in their numerical results is that for some simply supported FG plates (e.g., aluminum oxide/Ti–6Al–4V FG plates), there is no buckling point and what takes place is only a large deflection phenomenon. They also showed that for the homogenous plates the fundamental frequency vanishes as the temperature approaches the critical point but fundamental frequencies of the FG plates do not vanish. This shows that for simply supported FG plates, eigenvalue analysis for buckling study, used by many researchers as [11–17], will yield erroneous results. Thermal postbuckling analysis of simply supported, shear deformable geometrically mid-plane symmetric FG plates under thermal loading was investigated by Shen [18]. He analyzed perfect and imperfect FG plates under different sets of loading conditions and showed that for the case of heat conduction, the postbuckling path for geometrically perfect mid-plane symmetric FG plates is no longer of the bifurcation type.

In this paper, an exact solution is developed for nonlinear analysis of shear deformable FG plates under transverse and in-plane tensile and compressive loadings. By using the von Karman nonlinear strains and gradual variation of material properties, the nonlinear equilibrium equations are obtained and then reduced to a linear differential equation. This equation is solved for simply supported BCs and the effects of many parameters such as volume fraction index, loading type and plate dimensions are studied. It is shown that the eigenvalue buckling analysis of simply supported FG plates is physically incorrect. Also, the results show that the linear analysis yields wrong results not only quantitatively but also qualitatively.

2. Theoretical formulation

2.1. Displacement field and strains

An FG plate of uniform thickness h and length $l=2a$, which is made of a ceramic at the upper surface and a metal at the lower surface, is considered in this study. The displacement field based on the FSDT is as follows:

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\phi_x(x,y),\\ v(x,y,z) &= v_0(x,y) + z\phi_y(x,y),\\ w(x,y,z) &= w_0(x,y), \end{aligned} \tag{1}$$

where (u, v, w) are the displacements in the (x, y, z) directions, respectively, with z in the thickness direction and origin at the mid-plane. Also, (u_0, v_0, w_0) are the displacements of the mid-plane in the (x, y, z) directions, respectively, and ϕ_x and ϕ_y are the section normal vector rotations about the y-, and x-axes, respectively. The nonlinear von Karman strain–displacement relations are used as follows:

$$\begin{aligned} \varepsilon_{xx} &= u_{0,x} + \tfrac{1}{2}w_{,x}^2 + z\phi_{x,x},\\ \varepsilon_{yy} &= v_{0,y} + \tfrac{1}{2}w_{,y}^2 + z\phi_{y,y},\\ \varepsilon_{zz} &= 0,\\ \gamma_{xy} &= u_{0,y} + v_{0,x} + w_{,x}w_{,y} + z(\phi_{x,y} + \phi_{y,x}),\\ \gamma_{yz} &= \phi_y + w_{,y},\\ \gamma_{xz} &= \phi_x + w_{,x}, \end{aligned} \tag{2}$$

where ε_{xx}, ε_{yy}, and ε_{zz} are the normal strains in the x, y, and z directions, respectively, and γ_{xy}, γ_{yz}, and γ_{xz} are the shear strains. Also, the subscript comma denotes differentiation w.r.t. the variable followed by it.

2.2. FG modeling

The effective material property, P_{eff}, of the FG plate is assumed to vary through-the-thickness of the plate as a function of the volume fractions of the constituents. The volume fraction of the ceramic is related to that of the metal in the following way (see [19,1]):

$$V_m + V_c = 1, \tag{3}$$

where subscripts m and c denote metal and ceramic, respectively. If the upper surface of the FG plate were ceramic-rich, then V_c would be defined using a simple power law function as follows:

$$V_c = \left(\frac{2z+h}{2h}\right)^n, \tag{4}$$

where n is the volume fraction index. It is assumed that in the FG plates under consideration, the matrix phase is reinforced by spherical particles of a particulate phase. The Mori–Tanaka scheme (see [19,20]) is applicable to regions of the graded microstructure, which has a well-defined continuous matrix and a discontinuous particulate phase for estimating Young's modulus, E, and Poisson's ratio, ν. The effective local bulk modulus, K, and the shear modulus, G, obtained by the Mori–Tanaka scheme for a random distribution of isotropic particles in an isotropic matrix, are expressed as (see [21,22,23])

$$\frac{K-K_m}{K_c-K_m} = \frac{V_c}{(1+(1-V_c)((K_c-K_m)/(K_m+\frac{4}{3}G_m)))}, \tag{5}$$

$$\frac{G-G_m}{G_c-G_m} = \frac{V_c}{(1+(1-V_c)((G_c-G_m)/(G_m+f_1)))}, \tag{6}$$

where

$$f_1 = G_m\frac{9K_m+8G_m}{6(K_m+2G_m)}, \tag{7}$$

2.3. Constitutive relations

The constitutive relations can be expressed as

$$\begin{Bmatrix} \sigma_{xx}\\ \sigma_{yy}\\ \sigma_{yz}\\ \sigma_{xz}\\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0\\ Q_{12} & Q_{22} & 0 & 0 & 0\\ 0 & 0 & kQ_{44} & 0 & 0\\ 0 & 0 & 0 & kQ_{55} & 0\\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy} \end{Bmatrix}, \tag{8}$$

where σ_{xx} and σ_{yy} are normal stresses in x and y directions, respectively. Also, σ_{yz}, σ_{xz}, and σ_{xy} are shear stresses and k is the shear correction factor set equal to 5/6 in this paper (see [21]). Through-the-thickness variable stiffness coefficients, Q_{ij}, are defined by

$$\begin{aligned} Q_{11} &= Q_{22} = \frac{E(z)}{1-\nu(z)^2},\\ Q_{12} &= \frac{\nu(z)E(z)}{1-\nu(z)^2},\\ Q_{44} &= Q_{55} = Q_{66} = \frac{E(z)}{2[1+\nu(z)]}. \end{aligned} \tag{9}$$

2.4. Equations of motion

Use of Hamilton's principle yields the Euler–Lagrange equations as (see [24,25])

$$\begin{aligned} N_{x,x} + N_{xy,y} &= 0,\\ N_{xy,y} + N_{y,y} &= 0, \end{aligned}$$