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Shear deformation of voids with contact modelled by internal pressure

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ABSTRACT

The behaviour of voids in a ductile material subject to simple shear or to a shear-dominated stress state is analyzed numerically. Here the stress triaxiality is so low that instead of void volume growth to coalescence there is void closure leading to micro-cracks that rotate in the shear field. At some stage of the deformation, the void surfaces will come in contact so that sliding with or without friction will start to occur. To avoid problems with strong mesh distortion in the large strain field around the deforming void and with mesh resolution at the tip of the crack, an internal pressure is applied as an approximate representation of void surfaces pressed together in frictionless sliding, and also remeshing is applied. This micromechanical model for a strain hardening elastic–plastic material shows that a maximum overall shear stress is reached, at which localization of plastic flow occurs, leading to final failure in the material.

1. Introduction

Ductile fracture in metals occurs mainly by the growth of microscopic voids until they coalesce with neighbouring voids to initiate the formation of macroscopic cracks. When the voids grow under high stress triaxiality, as is often the case in ductile fracture situations, they remain near spherical and in such cases the ductile fracture process is rather well described by porous ductile material models as that proposed by Gurson [1]. Many numerical cell-model studies or experimental studies for this type of behaviour leading to ductile fracture have been carried out [2,3]. When the stress triaxiality is lower the voids will tend to elongate in the tensile direction and will deviate more and more from the circular cross-section the lower the stress triaxiality. A porous ductile material model accounting for such ellipsoidal cavities has been developed by Gologanu et al. [4]. However, when there is no hydrostatic tension in the material no void growth is predicted at all, but still ductile failure is observed in situations of pure shear. Recently, Nahshon and Hutchinson [5] have proposed an extension of the Gurson model to also describe failure in pure shear, but here the damage parameter f is no longer a geometrically well-defined void volume fraction, so that the model is more like continuum damage mechanics.

The behaviour of voids under shear has been investigated by a number of authors. Thus, for an initially spherical void in a linearly viscous material under remote shearing, Fleck and Hutchinson [6] have found that the void becomes spheroidal, rotates, and finally forms a penny-shaped crack. Fleck et al. [7] have analyzed an elastic–plastic shear specimen containing a row of circular cylindrical voids, to model experiments of Cowie et al. [8], in which the shear loading was combined with tension or compression and failure tended to occur by shear localization and void-sheet fracture in the direction parallel to the shear loading. Some of these finite strain analyses for perfect plasticity account for contact with an inclusion inside the void, from which the void has originally nucleated. More recently, Barsoum and Faleskog [9] have carried out full three-dimensional (3D) analyses for similar shear specimens containing spherical voids in order to model their experiments [10] on ductile fracture in a double-notched tube specimen loaded in combined tension and torsion. The behaviour of initially spheroidal voids in a shear field have been analyzed by Scheyvaerts et al. [11] for different initial orientations of the spheroid, relative to the direction of shear, and 3D analyses for voids in shear fields have also been carried out by Leblond and Mottet [12]. Relevant to these studies of voids in a shear field are also analyses of Anderson et al. [13], considering the effect of a row of micro-cracks in a material subject to shear, where it is shown that localization can result from crack rotation and stretching, even when strain hardening occurs.

Several experimental and theoretical investigations have considered voids in tensile test specimens, where final void-sheet failure involves significant shearing as the voids grow to coalescence in a shear band inclined to the tensile direction [14–18]. Characteristic of these studies is that besides shear the voids are subjected to sufficiently high stress triaxiality, so that the void volumes grow during shearing.

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The purpose of the present study is to find a micromechanically based mechanism of ductile failure in shear under low stress triaxiality, which can rely on void deformations but cannot rely on a growing void volume fraction, since the voids close up. The plane strain shear specimen of Fleck et al. [7] is reconsidered, but the analyses are carried much further. Now strain hardening is accounted for and the focus is on studying situations where the void closes up to form a crack or something like a crack, which then rotates and stretches in a way somewhat analogous to the cracks analyzed by Anderson et al. [13]. In a full finite strain analysis of void deformations, the formation of a completely closed crack with crack surface contact and with much material flow around the crack tips presents a significant complication. In the present analyses these complications are accounted for in an approximate manner, by avoiding that the cracks close completely. Instead, an internal hydrostatic pressure is applied to simulate the crack surface contact, but the pressure level is computed such that the aspect ratio of the flattened void does not pass a specified limiting value. In this way, it is demonstrated that the voids developing into shear cracks do result in an overall softening behaviour of the material.

2. Problem formulation

As in Fleck et al. [7] a shear specimen is analyzed, containing a periodic array of voids (Fig. 1), corresponding to conditions of simple shear. The analyses are carried out for plane strain conditions and the voids are initially circular cylindrical with radius R_0. The initial height of the shear specimen is $2B_0$ and the initial spacing between void centres is $2A_0$. Finite strains are accounted for and the analysis is based on a convected coordinate Lagrangian formulation of the field equations, with a Cartesian x^i coordinate system used as reference. Here, g_{ij} and G_{ij} are metric tensors in the reference configuration and the current configuration, respectively, with determinants g and G, and $\eta_{ij} = 1/2(G_{ij}-g_{ij})$ is the Lagrangian strain tensor. The contravariant components τ^{ij} of the Kirchhoff stress tensor on the current base vectors are

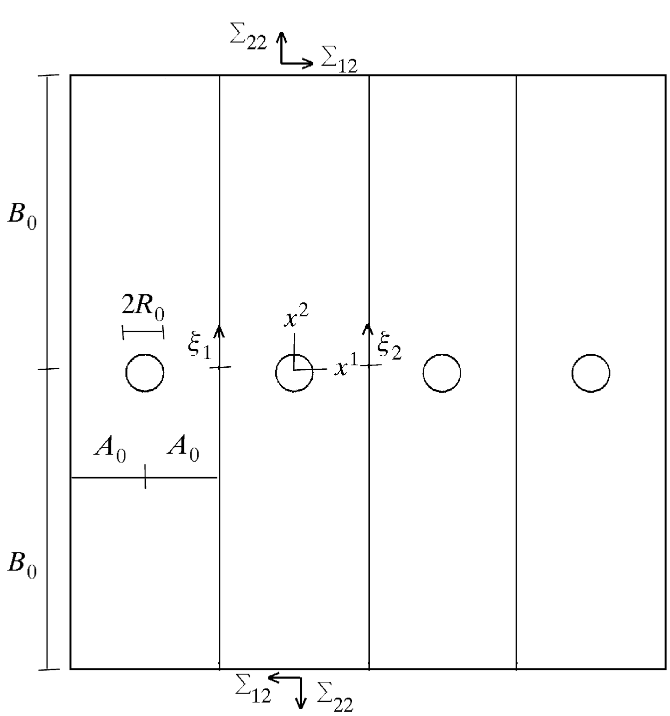

Fig. 1. Periodic array of cylindrical voids used to model ductile failure in shear.

related to the components of the Cauchy stress tensor σ^{ij} by $\tau^{ij} = \sqrt{G/g}\sigma^{ij}$. Then, in the finite strain formulation for a J_2 flow theory material with the Mises yield surface the incremental stress–strain relationship is of the form $\dot{\tau}^{ij} = L^{ijk\ell}\dot{\eta}_{k\ell}$. The instantaneous moduli are specified in [19,20]. The true stress-logarithmic strain curve in uniaxial tension is taken to follow the power law

$$\varepsilon = \begin{cases} \sigma/E, & \sigma \le \sigma_Y \\ (\sigma_Y/E)(\sigma/\sigma_Y)^{1/N}, & \sigma \ge \sigma_Y \end{cases} \tag{1}$$

with Young's modulus E, the initial yield stress σ_Y and the power hardening exponent N. Furthermore, the instantaneous moduli make use of Poisson's ratio v, and of the Mises stress $\sigma_e = (3s_{ij}s^{ij}/2)^{1/2}$, where s^{ij} is the stress deviator, and the tangent modulus E_t, which is the slope of the uniaxial stress–strain curve in Eq. (1) at the stress level σ_e.

The boundary conditions on the top and the bottom of the shear specimen are specified by

$$\dot{u}^1 = \dot{U}_I, \ \ \dot{u}^2 = \dot{U}_{II} \quad \text{for} \quad x^2 = B_0 \tag{2}$$

$$\dot{u}^1 = -\dot{U}_I, \ \ \dot{u}^2 = -\dot{U}_{II} \quad \text{for} \quad x^2 = -B_0 \tag{3}$$

where $\dot{U}_I$ and $\dot{U}_{II}$ are constants, $\dot{U}_I$ is prescribed, and $\dot{U}_{II}$ is calculated such that the stress ratio on the top surface has the prescribed value

$$\Sigma_{22}/\Sigma_{12} = \kappa \tag{4}$$

The average stresses on the top surface are calculated as

$$\Sigma_{22} = \frac{1}{2A_0}\int_{-A_0}^{A_0} T^2 \, dx_1, \quad \Sigma_{12} = \frac{1}{2A_0}\int_{-A_0}^{A_0} T^1 \, dx_1$$
$$\text{for} \quad x^2 = B_0 \tag{5}$$

where T^i are the contravariant components of the nominal surface tractions and u_i are the displacement components. When only the region around one void is analyzed numerically, i.e. the region for $-A_0 \leqslant x^1 \leqslant A_0$, periodicity conditions have to be satisfied on the two sides

$$u^1(\xi_1) = u^1(\xi_2), \quad u^2(\xi_1) = u^2(\xi_2) \tag{6}$$

$$T^1(\xi_1) = -T^1(\xi_2), \quad T^2(\xi_1) = -T^2(\xi_2) \tag{7}$$

where ξ_1 and ξ_2 are length measuring coordinates defined in Fig. 1.

With the elongated rectangular region considered here, the focus is on studying the interactions in a single row of voids along the x^1-axis. If there were also periodic voids in the x^2-direction, the cell would have to end at symmetry planes between voids, and the simple boundary conditions in Eqs. (2) and (3) would be replaced by periodicity conditions. This, however, would not be able to represent the development of a single shear band as that found in the present analyses, resulting from the material imperfection due to the single row of voids. It is noted that in the present analyses for the elongated rectangular region, the overall lateral strain remains zero so that the lateral stress is non-zero, and the triaxiality will not remain constant in time.

The model here makes use of a pressure load inside the void that starts during the calculation and subsequently is increased. It is emphasized that this does not occur in reality, but is used to model the contact forces on the surface of a collapsed void. The possibility of friction cannot be modelled this way, so friction is neglected throughout the present study.

The void surface, $(x^1)^2+(x^2)^2 = R_0^2$, is initially stress free so that $T^1 = T^2 = 0$. However, at a later stage of the deformation, a hydrostatic pressure p is applied inside the voids to simulate the effect of crack surface contact in a relatively simple manner. When