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Free vibration of antisymmetric angle-ply-laminated plates including transverse shear deformation: Spline method

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ABSTRACT

A free vibration study of antisymmetric angle-ply composite plates including shear deformation and rotatory inertia using the point collocation method and applying spline function approximations is presented. The equations of motion for the plate are derived using the theory of Yang, Norris and Stavsky. The solution is assumed in a separable form to obtain a system of coupled differential equations in displacement and rotational functions and these functions were approximated by Bickley-type splines of order three. A generalized eigenvalue problem is obtained and solved numerically for an eigenfrequency parameter and an associated eigenvector of spline coefficients. The vibrations of two- and four-layered plates, made up of several types of layer materials and subjected to two types of boundary conditions are considered. Parametric studies were made of the variation of frequency parameters with respect to the aspect ratio, side-to-thickness ratio and ply angle. The numerical results are presented through diagrams and, in some cases, are compared with results obtained by FEM.

1. Introduction

Composite structural elements play a significant role in several fields including aerospace, automobile and ship building, because of their more desirable damping and shock absorbing characteristics than those of homogeneous ones. In composite plates, the influence of shear deformation becomes significant as the plate thickness increases and hence theories incorporating this aspect are highly desired, along with suitable numerical techniques.

Several types of theories have been developed to treat the mechanical behavior of composite laminates. Classical laminate plate theory (CLPT) [1] due to Kirchoff and Love neglect shear deformation giving inaccurate results for moderately thick plates. The frequencies calculated by using CLPT are generally higher than those obtained by Mindlin plate theory[2]. Shear deformation theories have been proposed by many researchers among which the first theory for laminated isotropic plates was due to Stavsky [3]. This has been generalized to laminated anisotropic plates by Yang et al. [4] as the YNS theory. Sun and Whitney [5] and Whitney and Pagano [6] also discussed the YNS theory for laminated plates consisting of an arbitrary number of bonded anisotropic layers. Bert and Chen [7] presented a closed-form solution for the free vibration of simply supported rectangular plates of antisymmetric angle-ply laminates. Reddy [8] and Ghosh and Dey [9] presented a FEM solution for laminated plates using YNS and higher order theories, respectively. Recently, Ferreira [10] and Kabir et al. [11] used different methods for the analysis of laminated composite plates. Recently, Viswanathan and Sang-Kwon Lee [12] discussed the problem of vibration of cross-ply-laminated plates including shear deformation theory using spline function techniques.

This paper deals with the free vibration of antisymmetric angle-ply-laminated composite plates including shear deformation using a method of collocation with splines. The same, as well as different types of materials are used in different layers. The problem is formulated using the YNS theory from which is obtained a system of coupled differential equations on a set of assumed displacement and rotational functions, which are functions of a space co-ordinate. A spline technique was used, which was chosen over a number of other methods available for such problems, like those of Galerkin, Runge–Kutta, Frobenius, Chebyshev collocation and FEM. The choice of this method is due to the possibility that a chain of lower order approximations, as used here, can yield greater accuracy than a global high-order approximation [13]. The spline strip method incidentally, was used by Mizusawa and Kito [14] to study the vibration of cross-ply-laminated cylindrical panels. Bickley [13] successfully tested the spline collocation method over a two-point boundary value problem with a cubic spline. Soni and Sankara Rao [15], Irie et al. [16], Irie and Yamada [17], Navaneethakrishnan and

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Chandrasekaran [18], Navaneethakrishnan [19], Navaneethakrishnan et al. [20] and Viswanathan and Navaneethakrishnan [21,22] have also successfully applied this method, but most of them used only a single spline function in their problems.

In this work, three displacement functions and two rotational functions were approximated by splines, which are cubic, in a system of coupled equations. The convergence characteristics were revealed. These splines are simple and clear for analytical process and have significant computational advantage. Even on theoretical consideration spline functions are more elegant and convenient to conceive, construct and handle, as approximating, interpolating and curve fitting functions than many others. It is well known that polynomial approximations are always possible (Weierstrass' Theorem) and of practical use over a given set of points. Spline is not only a polynomial approximation, but also a generalized polynomial in the sense that it is a piecewise polynomial that can be made as smooth as required, at the junction points. If the approximate solution is for a boundary (or, initial) value problem, comprised of a differential equation of order k, the order of the spline can be limited to k+1 and not $n-1$, where n is the number of points over which the solution curve is approximated, with $n \gg k$. It is elegant since, to start with, the function is assumed in its final form, with only the coefficients to be determined; and only differentiations (as against integrations) are carried out to make it satisfy the boundary value problem and the associated boundary conditions (which may involve differential coefficients of order $<=k$).

Assuming the solution in a separable form, a system of coupled differential equations in displacement and rotational functions is obtained and these functions are approximated by Bickley-type splines of order three. Collocation with these splines yielded a set of field equations which, along with the equations of boundary conditions, reduce it to a system of homogeneous simultaneous algebraic equations on the assumed spline coefficients. Thus resulting generalized eigenvalue problem is solved for a frequency parameter, using eigensolution technique, to obtain as many frequencies as required, starting from the least. From the eigenvectors the spline coefficients are computed from which the mode shapes and shear rotations can be constructed.

Parametric studies are made of the variation of frequency parameters with respect to the aspect ratio and side-to-thickness ratio for two and four layers and the ply angle for four layers. Three different layer materials were considered. Significant mode shapes were obtained. Numerical results are presented in terms of graphs and tables and discussed.

2. Formulation of the problem

Consider a plate of length a, width b and constant thickness h composed of an even number of thin layers of equal thickness made up of anisotropic materials bonded together, with an orientation angle of θ and $-\theta$. The xy-plane (reference surface) is placed at mid-depth of the plate, while the z-axis is normal to it. Following Bert and Chen [7] and Reddy [8], the displacement components based on YNS theory are assumed to be

$$u = u_0(x,y,t) + z\psi_x(x,y,t),\ v = v_0(x,y,t) + z\psi_y(x,y,t),\ w = w(x,y,t) \tag{1}$$

where u, v and w are the displacement components in the x, y and z directions, respectively, u_0 and v_0 are the in-plane displacements of the middle plane and ψ_x and ψ_y are the shear rotations of any point on the middle surface of the plate.

The displacement components u_0, v_0, w and shear rotations ψ_x and ψ_y are assumed in the form

$$\begin{aligned} u_0(x,y,t) &= u(x,y)e^{i\omega t} \\ v_0(x,y,t) &= v(x,y)e^{i\omega t} \\ w(x,y,t) &= w(x,y)e^{i\omega t} \\ \psi_y(x,y,t) &= \psi_y(x,y)e^{i\omega t} \\ \psi_x(x,y,t) &= \psi_x(x,y)e^{i\omega t} \end{aligned} \tag{2}$$

where ω is the angular frequency of vibration and t is the time.

Using Eq. (2) in the constitutive equations and the resulting expressions for the stress and moment resultants in the equation of motion [8], the governing equations of motion in u, v, w, ψ_x and ψ_y are obtained in the matrix form

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} & L_{14} & L_{15} \\ L_{21} & L_{22} & L_{23} & L_{24} & L_{25} \\ L_{31} & L_{32} & L_{33} & L_{34} & L_{35} \\ L_{41} & L_{42} & L_{43} & L_{44} & L_{45} \\ L_{51} & L_{52} & L_{53} & L_{54} & L_{55} \end{bmatrix} \begin{bmatrix} u \\ v \\ \psi_x \\ \psi_y \\ w \end{bmatrix} = \{0\} \tag{3}$$

where $L_{ij} = L_{ji}$ are linear differential operators in x and y, given by

$$\begin{aligned} L_{11} &= A_{11}\frac{\partial^2}{\partial x^2} + A_{66}\frac{\partial^2}{\partial y^2} + p\omega^2,\ L_{12} = (A_{12} + A_{66})\frac{\partial^2}{\partial x \partial y} \\ L_{13} &= 2B_{16}\frac{\partial^2}{\partial x \partial y} \\ L_{14} &= B_{16}\frac{\partial^2}{\partial x^2} + B_{26}\frac{\partial^2}{\partial y^2},\ L_{22} = A_{66}\frac{\partial^2}{\partial x^2} + A_{22}\frac{\partial^2}{\partial y^2} + p\omega^2 \\ L_{23} &= B_{16}\frac{\partial^2}{\partial x^2} + B_{26}\frac{\partial^2}{\partial y^2} \\ L_{24} &= 2B_{26}\frac{\partial^2}{\partial x \partial y},\ L_{33} = D_{11}\frac{\partial^2}{\partial x^2} + D_{66}\frac{\partial^2}{\partial y^2} - KA_{55} + I\omega^2 \\ L_{34} &= (D_{11} + D_{66})\frac{\partial^2}{\partial x \partial y} \\ L_{35} &= -KA_{55}\frac{\partial}{\partial x},\ L_{44} = D_{66}\frac{\partial^2}{\partial x^2} + D_{22}\frac{\partial^2}{\partial y^2} - KA_{44} + I\omega^2 \\ L_{45} &= -KA_{44}\frac{\partial}{\partial y} \\ L_{55} &= -KA_{55}\frac{\partial^2}{\partial x^2} - KA_{44}\frac{\partial^2}{\partial y^2} - p\omega^2 \text{ and } L_{15} = L_{25} = 0 \end{aligned} \tag{4}$$

Here p and I are the normal and rotary inertia coefficients defined by

$$(p, I) = \int \rho^{(k)}(1, z^2)\,dz \tag{5}$$

where $\rho^{(k)}$ is the material density of the k-th layer and K is the shear correction coefficient. The value of K for a general laminate depends on the lamina's properties and lamination scheme, and may be calculated by various static and dynamic methods [4, 23–26].

Let the edges $y = 0$ and b of the plate be simply supported. Then the displacements and rotational functions are assumed in the separable form as

$$\begin{aligned} u(x,y) &= U(x)\cos\frac{n\pi y}{b} \\ v(x,y) &= V(x)\sin\frac{n\pi y}{b} \\ w(x,y) &= W(x)\sin\frac{n\pi y}{b} \\ \psi_y(x,y) &= \Psi_y(x)\cos\frac{n\pi y}{b} \\ \psi_x(x,y) &= \Psi_x(x)\sin\frac{n\pi y}{b} \end{aligned} \tag{6}$$