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Meshfree simulation of concrete structures and impact loading

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ABSTRACT

The impact resistance of concrete structures is of major importance in engineering application. Computational methods are increasingly used for such types of applications but face difficulties due to the complex physical behaviour involving large deformations and large strains. Meshfree methods seem ideally suited to deal with these types of problems. In this manuscript, we present stochastic simulations based on the element-free Galerkin method to predict upper and lower bounds of the impact resistance of concrete structures. We account for stochastic distribution of material parameters and validate our results with benchmark experiments conducted by the group of Hanchak.

1. Introduction

Computational modelling of concrete under impact loading remains one of the key challenges in civil engineering. Besides computational methods, constitutive models are an important ingredient of any mechanical model. For high dynamic loading it is important to accurately capture the material response under extreme pressure loading and the so-called strain rate effect. Popular constitutive models based on damage mechanics include the work in references 1–4. In reference 5, the authors extended the famous Johnson-Cook (JC) [6] model to concrete materials; the JC model accounts for strain rate and temperature effects and also plastic deformations. A quasi-continuum plasticity approach capturing the dynamic buckling strength of sandwich structures was proposed by Rabczuk et al. [7]. Coupled damage-plasticity models were proposed for instance in references 8–13. In this work, we employ a constitutive model proposed in reference 14. It employs a dynamic damage variable that delays the damage evolution in order to take the strain-rate effect into account. This dynamic damage variable depends on previous damage increments and the associated damage rates. In reference 15, the authors extended their scalar damage model for isotropic damage to anisotropic damage by introducing a vectorial damage.

Another important ingredient to model the impact resistance of concrete is the computational method. Many studies are baed on finite element analysis [16–24]. Often, element-deletion methods were exploited in order to allow for large deformations and complete perforations [25,26]. Meshfree or meshless methods are good alternatives to FEM [27–37] as they can model large deformation and perforations without additional techniques and much loss of accuracy. An overview and computer implementation aspects of meshfree methods (MM) is discussed in reference 38. MM have also been used to model impact events. For example, the authors in reference 39 accurately predicted the penetration depth and residual impact velocities compared to the experiments done by the authors in reference 40. These authors reported finite element simulations underpredict the impact resistance of concrete. While dynamic fracture in MM was initially captured quite naturally by separation of nodes [41–47], the authors in reference 32 and 48 for example found that such an approach might lead to numerical fracture. More sophisticated approaches such as XFEM (extended finite element method) [49,50] or smoothed extended finite element method [51–53], extended MM [36,54–63], extended isogeometric analysis formulations [64,65], the phantom node method [66–69] or smoothed phantom node approaches [70], recent multiscale methods [71–75] or efficient remeshing techniques [76–82] might also be applied to dynamic fracture [83,84]. However, while they seem well suited to capture a moderate number of propagating cracks, their performance to capture a large number of cracks in a large deformation setting still needs to be shown. A compromise to above mentioned approaches is the cracking particles method (CPM) [85,86]. In the CPM, fracture is modelled by set of cracked particles. Several improvements have been incorporated into the CPM [87–92]. Since MM are computational costly, they have been coupled to finite element methods [30,92]. The two mentioned formulations have also been applied to predict the impact resistance of concrete structures.

The majority of the publications (see the list above) are focused on deterministic approaches but it is well known that this can lead to unrealistic crack patterns as the ones predicted in reference 39.

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In these simulations, cracks are too close to each other. Stochastic approaches such as introducing some randomness in the tensile strength [93] can alleviate this unrealistic behaviour [94–96]. However, none of these simulations consider stochastic material parameters though it is barely possible to calibrate the material parameters uniquely and exactly.

On the other hand, every computational method needs to be validated. Classical benchmark problems for impact resistance of concrete include the experiments by references 97–103. The experiment exploited in this manuscript was carried out by the authors in reference 104. In these experiments, concrete specimens were subjected to impactors with various velocities.

In summary: We present stochastic simulations to predict the impact resistance of concrete. The element-free Galerkin (EFG) method is exploited in combination with a viscous damage-plasticity model [14]. A simple node splitting algorithm described in reference 89 has been exploited in order to avoid artificial fracture. It can be considered as a special case of the CPM. Finally, our simulations are validated by comparison to experimental data from our own laboratory and from Hanchak et al. [104].

2. Governing equations and discretization

We solve the equation of motion that can be stated in weak form by

$$\begin{aligned}\delta W_{int} - \delta W_{ext} - \delta W_{kin} &= 0\\ \delta W_{int} &= \int_\Omega \delta\epsilon_{ij} : \sigma_{ij}\, d\Omega\\ \delta W_{ext} &= \int_{\Gamma_t} \delta u_i t_i\, d\Gamma + \int_\Omega \delta u_i b_i\, d\Omega\\ \delta W_{kin} &= \int_\Omega \rho u_i \ddot{u}_i\, d\Omega\end{aligned} \tag{1}$$

δW_{kin}, δW_{ext} and δW_{int} indicating the kinetic energy, external work and internal energy, respectively; Ω is the domain and $\Gamma_t \cup \Gamma_u = \Gamma$, with $\Gamma_t \cap \Gamma_u = 0$ is the external boundary consisting of traction and displacement boundary conditions indicated by the subscript t and d, respectively. The components of the linear strain tensor is denoted by ϵ_{ij} and σ_{ij} are the components of the Cauchy stress tensor; the components of the traction and body force vector are given by t_i and b_i, respectively; u_i are the components of the displacement field, ρ is the density and the superimposed 'dot' stands for material time derivatives. As already mentioned in the introduction, we employ the EFG method [105] to discretize the displacement field. It can be shown that the EFG approximation is given by

$$\mathbf{u}(\mathrm{x}, t) = \sum_{I \in S} N_I(\mathrm{x})\mathbf{u}_I(t) \tag{2}$$

$\mathbf{u}_I$ being the nodal parameters of the displacement field, which are unequal to the physical displacement values at that point, or in other words $\mathbf{u}(\mathbf{x}_I) \neq \mathbf{u}_I$. The shape functions are denoted by $N_I(\mathbf{x})$. They are obtained from miminization of a discrete L_2 norm which finally leads to:

$$\mathbf{N}(\mathbf{x}) = \mathbf{p}^T(\mathbf{x})\mathbf{A}^{-1}(\mathbf{x})\mathbf{B}(\mathbf{x}) \tag{3}$$

$\mathbf{A}(\mathbf{x}) = \sum_{I=1}^{n} w(\mathbf{X} - \mathbf{X}_I)\mathbf{p}(\mathbf{X}_I)\mathbf{p}^T(\mathbf{X}_I)$ is the moment-matrix and $\mathbf{B}(\mathbf{x})$ is computed as

$$\mathbf{B}(\mathbf{x}) = [w(\mathbf{x} - \mathbf{x}_1)\mathbf{p}(\mathbf{x}_1) \quad w(\mathbf{x} - \mathbf{x}_2)\mathbf{p}(x_2) \ldots w(\mathbf{x} - \mathbf{x}_n)\mathbf{p}(x_n)] \tag{4}$$

$w(\mathbf{X} - \mathbf{X}_I)$ denoting the weighting function and $\mathbf{p}^T(\mathbf{X})$ the polynomial basis. More details are given in reference 105. Exponential kernel function and linear basis functions are chosen, i.e. $\mathbf{p}^T(\mathbf{x}) = [1\, x\, y]$. Substituting the discretization, eq. (2), into the weak form of the equation of motion, eq. (1), leads to the well-known system of equations:

$$\mathbf{M}\ddot{\mathbf{D}} = \mathbf{F}_{ext} - \mathbf{F}_{int} \tag{5}$$

where the nodal parameters $\mathbf{u}_I(t)$ of the displacement field are stored in the global vector $\mathbf{D}$. It can be shown that the mass matrix is obtained by

$$\mathbf{M} = \int_\Omega \rho \mathbf{N}^T \mathbf{N}\, d\Omega \tag{6}$$

The external force vector and internal force vector is given by

$$\begin{aligned}\mathbf{F}_{ext} &= \int_\Omega \rho \mathbf{N}^T \mathbf{b}\, d\Omega + \int_{\Gamma_t} \mathbf{N}^T \bar{\mathbf{t}}\, d\Gamma\\ \mathbf{F}_{int} &= \int_\Omega \tilde{\mathbf{B}}^T \sigma\, d\Omega\end{aligned} \tag{7}$$

$\mathbf{N}$ and $\tilde{\mathbf{B}}$ being matrices containing the EFG shape functions and their spatial derivatives, respectively. We use a stabilized [48] nodally regularised [32] element-free Galerkin method [105]. We take advantage of the updated Lagrangian kernel formulation presented in reference 39 to ensure the stability of the method while simultaneously maintaining the applicability to extremely large deformations needed for dynamic fracture and fragmentation.

3. Constitutive model

The employed constitutive model is based on the approach presented by Rabczuk et al. [14]. While the original approach is a coupled damage-plasticity model, we removed the plasticity part from the formulation which reduces the number of material parameters. Subsequently, we summarize the basic equations of this constitutive model.

The strain rate $\dot{\epsilon}_{ij}$ is decomposed into an elastic part $\dot{\epsilon}^e_{ij}$ and a damage part $\dot{\epsilon}^d_{ij}$:

$$\dot{\epsilon}_{ij} = \dot{\epsilon}^e_{ij} + \dot{\epsilon}^d_{ij} \tag{8}$$

The stress-strain relation can be written as

$$\sigma_{ij} = (1 - D)\gamma C_{ijkl}\epsilon_{kl} \tag{9}$$

$D = D_S + D_D$ being a damage variable which is decomposed into a static part D_S and a dynamic part D_D, γ is a function accounting for high hydrostatic pressure response and C_{ijkl} denotes the components of the elasticity tensor. The formulation has been implemented in rate form as suggested in reference 14. We use the same damage surfaces in compression and tension

$$F_d = c_1 J_2^e + \kappa_d\left(c_2\sqrt{J_2^e} + c_3\epsilon^{(a)}_{e,max} + c_4 I_1^e\right) - \kappa_d^2 = 0 \tag{10}$$

c_i, $i = 1, \ldots, 4$ being material parameters, κ_d denotes the effective damage strain, I_1^e is the first invariant of the elastic strain tensor, J_2^e is the second invariant of the elastic strain tensor and $\epsilon^{(a)}_{e,max}$ indicates the a^{th} eigenvalue of the elastic strain tensor. A classical exponential function is adopted to model the damage evolution:

$$\begin{aligned}D_S &= 1 - e^{-\left(\frac{\kappa_d - e_0}{e_d}\right)^g} & \kappa_d &\geq e_0\\ D_S &= 0 & \kappa_d &< e_0\end{aligned} \tag{11}$$

where e_d, e_0 and g are material constants. A dynamic damage evolution is introduced decaying the static damage evolution. It is defined by

$$D_D = \int_{\tau=0}^{t} \frac{\partial D_S}{\partial t} H(t - \tau)\, d\tau \tag{12}$$