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Vibration and reliability of a rotating beam with random properties under random excitation

S.A.A. Hosseini, S.E. Khadem*

Department of Mechanical Engineering, Tarbiat Modarres University, Tehran, Iran

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Abstract

In this paper, vibration and reliability of a rotating beam with random properties under random excitations are studied. The rotating beam is under a stochastic load modeled as a stationary white noise. The cross-sectional area, elasticity modulus, moment of inertia, shear modulus, damping coefficient, mass density and rotational speed are modeled as random variables. To develop the equations of motion, the finite element method and space state analysis are applied. In order to consider the randomness of properties, a second order perturbation method is used. The effects of rotational speed, setting angle, hub radius, variances of random properties, correlation of random variables and damping matrix forms on the vibration and reliability of rotating beams, are studied completely.

Keywords: Vibration; Reliability; Rotating beam; Random properties; Random excitation

1. Introduction

The vibration analysis of a rotating beam is an important problem in understanding behavior of rotating structures such as propellers, turbine blades, helicopter rotors and satellite booms. To study vibration of rotating beams, Abbas [1] and Yokoyama [2] used finite element methods. Lee and Lin [3] studied the vibration of nonuniform rotating beam by neglecting the coriolis effect. Yang and Tsao [4] studied the vibration and stability of a pretwisted blade under non-constant rotating speed. Free out-of-plane vibration of a rotating beam with a nonlinear elastomeric constraint was investigated by Pohit et al. [5]. Al-Bedoore and Hamdan [6] developed a mathematical method for a rotating flexible arm undergoing large planar flexural deformations. Bazoune et al. [7] used finite element method to study the dynamic response of spinning tapered Timoshenko beams. Vibration of a rotating damped beam with an elastically restrained root was investigated by Lin et al. [8].

Aforementioned studies are limited to vibration of rotating beams with deterministic properties. However, in reality, properties of structures and mechanical systems are random. The stochastic finite element method (SFEM) was developed for analysis of a system with random properties by Combou [9] and Handa and Anderson [10]. Singh [11] studied the turbine blade reliability with random properties. Iwan and Haung [12] presented a procedure for obtaining the dynamic response of nonlinear systems with parameter uncertainty. Chakraborty and Dey [13] proposed a SFEM in the frequency domain for analysis of structural dynamic problems involving uncertain parameters. Ishida [14] studied the eigenvalue problem of uniform and optimum beam with uncertain cross-sectional area. Lin [15] evaluated the influence of the random parameter changes to the dynamic behavior of rotating Timoshenko beams. Also, Lin [16] investigated the probabilistic behavior of rotating Timoshenko beams. He used the finite difference method to find the sensitivity terms. Lately, present authors studied the free vibration of a rotating beam with random properties [17]. The properties were modeled as random fields. To study uncertainty, SFEM based on the second order perturbation method was applied. To discretize random fields, the three methods of midpoint, interpolation and local average were applied and compared.

*Corresponding author. Tel/fax: +98 21 88011001x3388.
E-mail address: Khadem@Modares.ac.ir (S.E. Khadem).

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Nomenclature

A	cross-sectional area
c	damping coefficient
E	elasticity modulus
$f_i (i = 1, \ldots, 8)$	Hermit shape functions
G	shear modulus
I	moment of inertia
K	shear correction factor
$[K_{YY}]$	conditional covariance matrix (covariance matrix of the nodal displacements of the rotating beam with random excitation and deterministic properties)
$[K_{ij}]$	elements of conditional covariance matrix $[K_{YY}]$
l	length of an element
L	length of the rotating beam
m	number of elements
R_0	hub radius
s_0	white noise intensity
Var	variances of random properties
w	transverse displacement
x'	element local coordinate
$y, \dot{y}$	displacement and velocity of the free end of the rotating beam
$\bar{y}$	threshold value of free-end displacement of rotating beam
ε_i	random variable with zero expectation
φ	bending rotation
$[\kappa_{YY}]$	total covariance matrix (covariance matrix of the nodal displacements of the rotating beam with random properties and random excitation)
$[\kappa_{ij}]$	elements of total covariance matrix $[\kappa_{YY}]$
ρ	mass density
σ_X	standard deviation of an arbitrary variable X
υ_{ij}	nondimensional parameter $(\upsilon_{ij} = \sqrt{(\kappa_{ij} - K_{ij})/K_{ij}})$
υ	parameter υ_{ij}, computed for the free-end displacement of the rotating beam
Ω	rotational speed
Ψ	setting angle

The main object in a rotating beam problem is the vibration analysis under external forces. In the early studies, the applied forces were modeled as deterministic excitations. But, in reality, the applied forces on the turbine blades are random excitations and should be analyzed by random vibration [18]. Ellishakoff et al. [19] studied the random vibration of beams with different boundary conditions. Also, Ellishakoff and Zhu [20] considered the random vibration of beams with the finite element method. Chen and Chen employed a finite element method to investigate the mean square response and reliability of metallic [21] and composite [22] rotating blade with external and internal damping effects under random excitation. Also, same authors [23] studied the random vibration and reliability of a damped composite rotating blade with flexural–torsional interaction under random excitation.

Analysis of systems with random properties under random excitations was considered in 1980s. The most important papers are as follows: Igusa and Der Kiureghian [24] investigated the reliability of structural systems with time invariant uncertain parameters subjected to stochastic excitations. Jensen and Iwan [25] studied the response of linear systems with random properties under random excitations. Chang and Yang [26] studied the response of a geometrically nonlinear beam with random properties under white noise. They used the time domain formulation and applied the local average method to discretize the random fields. Cherng and Wen [27] developed a method of stochastic finite element for obtaining response statistics and reliability of nonlinear-truss structure with uncertain system parameters under random excitation. Chakraborty and Dey [28] studied the stochastic structural response of uncertain system parameters with stochastic finite element simulation. Grundmann and Waubke [29] studied the nonlinear stochastic dynamics of systems with random properties under random excitation via statistical linearization method. Koyluoglu et al. [30] analyzed the two dimensional nonlinear frames with random properties under stationary random excitation. They used the frequency domain and applied the weighted integral method to discretized random fields. Lei and Qiu [31] proposed a procedure to derive the statistical characteristics of dynamic response of structures by using dynamic Neumann stochastic finite element method.

Generally, continuous systems with uncertainty in properties under random excitation can only be solved approximately [29]. The most important methods to deal with these problems are Monte-Carlo simulation (MCS) and SFEM. In this paper, for the first time, the vibration and reliability of a rotating beam with random properties under random excitation are studied. The rotating beam is under a stochastic force modeled as a white noise. The power spectral density of the force is uniform along the rotating beam. The cross-sectional area, elasticity modulus, shear modulus, moment of inertia, mass density, damping coefficient and rotational speed are modeled as random variables. Using the space state analysis and finite element method, the equations of motion for a rotating beam under random excitation are developed. Then, to consider the randomness of properties, the second order perturbation method is applied. The effects of rotational speed, setting angle, hub radius, variances of random properties, correlation of random properties and damping matrix are studied on the vibration and reliability of a rotating beam, completely.