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Free vibration analyses of simply supported beams carrying multiple point masses and spring-mass systems with mass of each helical spring considered

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Abstract

In the conventional finite element method (FEM), the dynamic characteristics of a longitudinally vibrating *rod* with mass density ρ_r, Young's modulus E_r, cross-sectional area A_r and total length ℓ_r are considered to be the same as those of a *helical spring* with stiffness constant $k_r = A_r E_r / \ell_r$ and total mass $m_r = \rho_r A_r \ell_r$. For a lumped-mass model, the mass matrix of a *rod* element is a 2×2 diagonal one with each of its non-zero coefficients to be equal to one half of the total rod mass (i.e., $0.5m_r$). Furthermore, the dynamic characteristics of a *rod* on the basis of last "lumped-mass" model have been found to be very close to those on the basis of "consistent-mass" model. Thus, one can easily take into account of the inertial effect of a *helical spring* using a massless one with "one half of its total mass", respectively, concentrated at its two ends (in Method 2) instead of modeling it by an elastic *rod* with uniform mass per unit length (in Method 1). When one more spring-mass system is attached to the beam, the total number of unknown constants increases "one" in Method 2 and "two" in Method 1, thus, Method 2 will reduce more effort than Method 1 for studying the dynamic behaviors of a beam carrying a number of spring-mass systems with mass of each *helical spring* considered. In this paper, the formulations of Methods 1 and 2 are presented first and then the numerical examples are illustrated to confirm the reliability of the presented theory and the developed computer programs. Finally, the effect concerning mass of each *helical spring* of the spring-mass systems is studied.

Keywords: Free vibration; Spring-mass system; Lumped-mass model; Consistent-mass model; Helical spring; Analytical method

1. Introduction

The literature concerning free vibration analyses of uniform or non-uniform beams carrying single or multiple spring-mass systems is plenty [1–13]. However, among the existing reports [1–13], most of them neglect the mass of *helical spring* of each spring-mass system except Refs. [10–13]. In Refs. [10,11], the frequency equation of a Bernoulli–Euler beam carrying a spring-mass system with inertial effect of the *helical spring* considered is derived first and then the influence of some non-dimensional factors on the non-dimensional fundamental frequency parameter is studied. In Refs. [12,13], the property matrices of a spring-mass system with inertial effect of its *helical spring* considered are determined first and then the conventional finite element method (FEM) is used to study the influence of masses of the *helical springs* on the dynamic characteristics of the entire vibrating system. In this paper, in addition to the fundamental frequency, the other lowest four natural frequencies and the lowest five mode shapes of the simply supported uniform beams carrying arbitrary number of point masses and spring-mass systems are determined with mass of each *helical spring* considered by using two analytical methods (Methods 1 and 2). In Method 1, each *helical spring* of the spring-mass systems is considered as an equivalent (continuous) *rod* with uniform mass per unit length, and in Method 2, each *helical spring* of the spring-mass systems is considered as a massless uniform *rod* with "one half" of total mass of the *helical*

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spring concentrated at two ends of the massless *rod*. It is noted that the conventional concept of adding "one third" of total mass of a *helical spring* to the lumped mass of a spring-mass system for taking into account of the inertial effect of a *helical spring* is correct only for a spring-mass system attached to a static (fixed) point (cf. Example 2.2-4 on p. 22 of Ref. [14]) rather than to a dynamic (vibrating) beam studied in this paper. In the conventional FEM [15,16], there exist two types of mass matrices for a *rod* element (or the other structural elements): consistent-mass matrix and lumped-mass matrix. Since the theories of the presented Methods 1 and 2 are similar to those of consistent-mass matrix and lumped-mass matrix, respectively, and the natural frequencies of a structural system based on the consistent-mass matrix are very close to the corresponding ones based on the lumped-mass matrix [17], it is under expectation that the lowest five natural frequencies and mode shapes of a beam carrying multiple spring-mass systems with mass of each *helical spring* considered obtained from Method 1 are very close to those obtained from Method 2.

In this paper, the formulations for Methods 1 and 2 are presented. Based on the algorithms of the two methods, the computer programs are developed and the numerical examples are illustrated. Furthermore, the conventional FEM is also used to solve the same problems. It has been found that the results of Method 1 and those of Method 2 are very close to each other, besides, the results of both Methods 1 and 2 are also very close to those of FEM. This confirms the reliability of the presented approaches and developed computer programs. Finally, the influence concerning mass of each *helical spring* on the dynamic behaviors of the simply supported uniform beams carrying multiple point masses and spring-mass systems is studied.

2. Formulation of Method 1

In Method 1, the *helical spring* of each spring-mass system attaching to the vibrating beam (see Fig. 1) is considered as a continuous *rod* with uniform mass per unit length, thus, each "spring-mass" system is also called a "rod-mass" system with each "helical spring" replaced by an equivalent (continuous) "rod" in this paper.

2.1. Equation of motion and displacement function of a beam segment

For free flexural vibration of the *i*th uniform beam segment (cf. Fig. 1), the equation of motion is given by

$$E_{bi}I_{bi}\frac{\partial^4 y_{bi}(x,t)}{\partial x^4}+\rho_{bi}A_{bi}\frac{\partial^2 y_{bi}(x,t)}{\partial t^2}=0 \quad (x_{i-1}\leqslant x\leqslant x_i), \tag{1}$$

where ρ_{bi}, E_{bi} and A_{bi} are mass density, Young's modulus and cross-sectional area of the *i*th beam segment, respectively, I_{bi} is the moment of inertia of area A_{bi}, while $y_{bi}(x,t)$ is the transverse displacement function of the *i*th beam segment at axial coordinate x (with origin 0 at left end of the beam) and time t.

For free vibration, one has

$$y_{bi}(x,t)=Y_{bi}(x)\mathrm{e}^{\mathrm{i}\omega t}, \tag{2}$$

where $Y_{bi}(x)$ is the amplitude function of transverse displacement function $y_{bi}(x,t)$ and ω is the natural frequency of the entire vibrating system as shown in Fig. 1.

Substituting Eq. (2) into Eq. (1), one has

$$Y''''_{bi}(x)-\beta^4_{bi}Y_{bi}(x)=0 \quad (x_{i-1}\leqslant x\leqslant x_i), \tag{3}$$

where the prime ($'$) denotes differentiation with respect to coordinate x and

$$\beta^4_{bi}=\omega^2\left(\frac{\rho_{bi}A_{bi}}{E_{bi}I_{bi}}\right). \tag{4}$$

The solution of Eq. (3) takes the form

$$Y_{bi}(x)=\bar{A}_{bi}\cos\beta_{bi}x+\bar{B}_{bi}\sin\beta_{bi}x + \bar{C}_{bi}\cosh\beta_{bi}x+\bar{D}_{bi}\sinh\beta_{bi}x \quad (x_{i-1}\leqslant x\leqslant x_i), \tag{5}$$

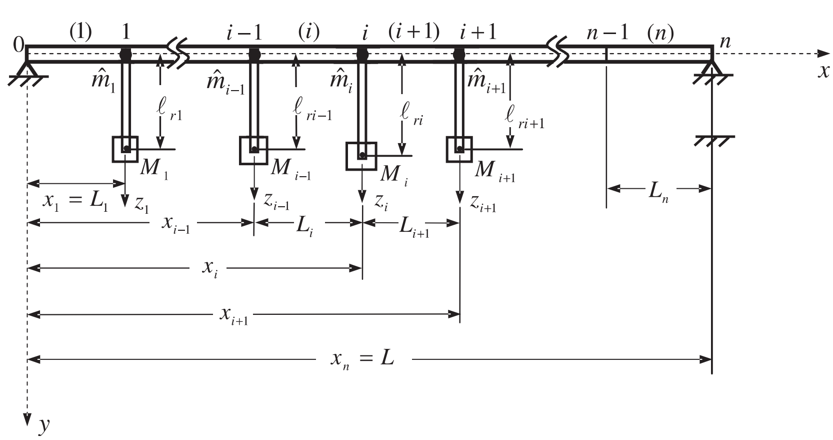

Fig. 1. A simply supported beam composed of n uniform beam segments (denoted by $(1),(2),\ldots,(i-1),(i),(i+1),\ldots,(n)$) separated by $n-1$ nodes (denoted by $1,2,\ldots,i-1,i,i+1,\ldots,n-1$) and, at each node, carrying a point mass $\hat{m}_i$ together with a pinned–pinned *rod* with length ℓ_{ri} and a tip (lumped) mass M_i, $i=1$ to $(n-1)$.