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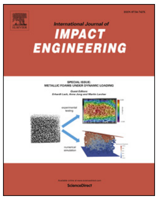

Progressive collapse of foam-filled conical frustum using kinematically admissible mechanism

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ABSTRACT

A kinematically admissible mechanism is presented for the progressive collapse of a foam-filled and unfilled circular frustum. The mechanism uses a three-limb model of collapse for the frustum shell, taking into account the plastic strain and circumferential strain energy dissipation during collapse. For the foam-filled case, the energy dissipation due to the plastic collapse of the foam and the interaction effects between it and outer frustum shell were considered. To account for the interaction effects, a pressure equivalent to the stress plateau of the foam was applied only to the inward bending proportion of the frustum. This is the result of our extensive experimental studies, which shows that Poisson's effects are negligible and that there will be no transverse strains resulting from the axial collapse of the frustum. We have further conducted analysis of empty frustum. A comparative analysis for the developed mechanism and previously conducted experimental studies showed good agreement between the two. Lastly, a parametric study of the mechanism correctly predicts that the magnitude of the crushing load depends on the length and proportion of the inward folds. Our upper-bound model gives a closed form solution for the collapse of foam-filled and unfilled frusta that can be used for crash energy management in automobiles.

1. Introduction

Vehicle crashworthiness manifests itself in the protection of car occupants and reduction of occupant fatalities, cargo and other road users, including pedestrians. Indeed, car safety is one of the major marketing incentives. Currently, numerous safety features exist in motor cars. These include frontal and rear energy absorbing systems, airbags, seats and seat belts, head rest, and crash avoidance systems. Current crash avoidance systems are in their infancy with many of the proposed engineering solutions address either minor road safety concerns, impractical to implement and/or rely on unproven technologies.

Undoubtedly, one of the most important safety features in a vehicle is an effective crash management system that is capable of absorbing the crash energy and reducing occupants' fatalities. Recent legislative pressure and competitiveness have led car manufacturers to design lightweight compact vehicles to reduce cost, fuel consumption & emissions. These requirements necessitate the use of light, economical, easy to fabricate and mount frontal crash management systems that make use of effective crash boxes to absorb the crash energy. This will alleviate the effect and consequences of frontal car crashes. A frontal crash box assembly is composed of front bumper beam (bumper rail), crash box containing two collapsible thin wall tubes made of a light alloy to absorb the majority of the impact energy, and longitudinal beams (side rails) which are connected to the car body. Fig. 1(a) shows a schematic of the three components.

In spite of the excellent efforts of the scientific community, the car companies and their original equipment manufacturers to develop effective crash management systems, there remain major challenges with the use of thin wall collapsible tubes and the accurate prediction of the collapse loads. For example, the use of uniform thin wall tubes can lead to (i) large crippling forces, (ii) transition from progressive collapse to global bending – especially for long tubes and oblique impact, and (iii) unstable collapse mode especially for very thin sections.

The purpose of this study is to replace thin wall tubes of uniform cross section with conical frusta. Conical frusta can be designed to overcome their counterparts' drawbacks. For example, they are capable of absorbing greater amounts of impact energy with reduced initial crippling force, leading to improved crashworthiness.

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The research community [1–7] has conducted considerable studies into the collapse behaviour of metallic thin wall tubes of mostly uniform cross-section, and to a much lesser degree into the fracture and fragmentation behaviour of fibre reinforced composites. In this study, we focus on the collapse behaviour of foam-filled and unfilled frusta. Specifically, the work focuses on the development of an upper bound closed from solution capable of describing the entire collapse process using kinematically admissible mechanism. The results of the closed form analysis are compared with experimental results of foam filled and unfilled frusta subject to collapse loads.

There exist a number of studies concerned with the numerical [8–13] and experimental [12–17] aspects for the collapse of both empty and foam-filled frustum. However, there have only been a few studies on the analytical modelling of collapse of foam-filled columns [2,5], and none known to the authors concerning foam-filled frustum.

Progressive collapse of thin walled columns or frusta can be viewed as repeated buckling of a shell. Timoshenko [18] first developed the associated theory for the buckling of cylindrical shells. The theories of elastic stability and the post-buckling deformation of cylinders under axial compressive loads were further developed by several authors, and were summarized in Ref. [19]. Alexander [20] developed an expression for the progressive concentric collapse of a thin-walled cylinder using three plastic hinges. Pugsley [21] expanded Alexander's work and developed an analytical expression for the mean load of collapse for diamond mode of collapse.

Based on the earlier work of Alexander and Pugsley, Abramowicz and Jones developed a solution for the static and dynamic collapse of square and circular columns [21–23]. However, the aforementioned models could only predict the mean collapse force instead of the instantaneous collapse force. Wierzbicki et al. [25] revisited Alexander's work and took an approach different from the concentric collapse of a circular column. They used a two-element model, with the first describing the fold and the second describing the transition region of the partial deformed zone where the next fold would form. Gupta and Velmurugan [26] decided to continue Alexander's work and proposed an analytical fold model for the instantaneous concentric collapse of circular columns. When compared with the experimental results, the model predicted the instantaneous force during the tube collapse quite well. However, it was not capable of predicting the instantaneous peak force. Subsequent papers took into account the variation of the circumferential strain [27], change in thickness and different yield strengths in tension and compression [28], and finally a three-limb model was proposed [29].

As previously mentioned, the theoretical progressive collapse of frustum has not been as thoroughly studied as the collapse of a thin wall tube of uniform cross section. Some of the earlier frustum models were developed by Mamalis et al. [17] for a circular frustum, and Reid and Reddy [15,30] for a square tapered tube. Their models predicted the mean collapse load. Gupta and Abbas [31] developed the first mathematical model to predict the instantaneous collapse load during the axisymmetric crushing of a thin-walled frustum. To account for the difference between the first and subsequent folds, a parameter was introduced that allowed the first fold to correlate well to the experimental results. A three-limb straight fold model by Hosseini et al. [32] improved on the previous work of Gupta by taking into account the variation of the circumferential strain, inside and outside fold length. Hosseini et al. also considered different yield strengths for compression and tension. A subsequent model by Hosseini et al. [33] considered the effect of the change of thickness in the tube during the crushing to better predict the proportion of the inward to outward fold. However, this only led to a minor improvement on the overall results.

Similarly, there has not been significant theoretical modelling of the progressive collapse of foam-filled columns. Reid et al. [34] developed an expression for the inclusion of the foam in a square column. The expression that was developed only took into account the effect of the densification of the foam on reducing the length of the fold. It was also determined that the interaction effect plays a significant role in the crushing of the tube. Therefore, Abramowicz [35] furthered this work by taking into account the coupling between the folds of the column and the compression of the foam. The theoretical model took into account the previous work of the tube collapse from Ref. [24], and included an energy dissipation term for the squeezing of the foam due to the inward bending of the shell. However, both of these studies focused only on the mean collapse load of the column but not the instantaneous. Some authors [36–38] have developed empirical relationships for the interaction of the foam and the shell for straight columns. However, these empirical models are dependent on the function for the form/shell interaction, which was determined numerically, thereby making these models invalid for a generalized solution.

Due to the absence of the research for the progressive collapse of foam-filled circular frusta, it is the objective of our paper to propose a three-limb straight-fold kinematic model accounting for the

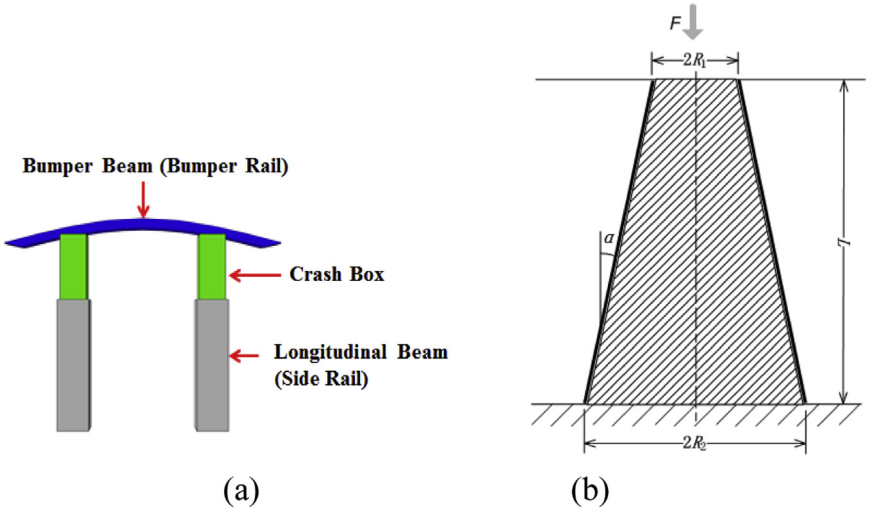

Fig. 1. (a) A schematic of an integrated frontal crash box system for automobile, and (b) Geometry of the foam filled frustum.