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A transport approach for analysis of shock waves in cellular materials

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ABSTRACT

This paper is concerned with a new approach for analytical modelling of shock-like behaviour encountered with high velocity crushing of cellular materials. The presented methodology is founded on integral transport equations which describe the response of a control volume moving inside another moving control volume and thus provides a technique for the direct handling of material discontinuities such as shock waves. The transport approach (being based on integration as opposed to differentiation) is shown to readily accommodate discontinuities in physical fields. Control volume movement is described by means of judiciously defined velocity fields that account for shock movement and domain distortion. A feature of the approach is the realisation of governing differential equations whose derivatives reflect the movement of the transporting control volume, which also serve as a means to establish the exactness of proposed solutions.

The transport approach is applied to investigate issues raised in the literature concerning discrepancies in results obtained from momentum and energy approaches. The ability of the transport approach to enforce global balances of energy and momentum reveal the correct approach to shock modelling in cellular materials. The efficacy of the method is demonstrated through its application to three case studies. The first two cases involve impact applied to the well-known Taylor-cellular bar and a rigid mass on a cellular bar; the results of which are shown to be exact agreement with the solutions provided in the literature. The third case study presents a new analytical solution for impact involving compacted material described with a Bingham plasticity model.

1. Introduction

Cellular materials are widely used in many energy absorber and blast protector structures due to their light weight, high specific strength and stiffness, high densification and advantageous constitutive behaviour [1,2]. The crushing of cellular materials is governed by localised progressive collapse of cells for relatively high striking velocities (i.e. velocities above a critical velocity). The deformation is localised at a relatively narrow band which is called a crush front [3–5]. This localised sharp crushing behaviour is similar to the propagation of a shock wave in continua which feature jumps in physical fields and state variables such as density, velocity, stress and temperature. Shock-like behaviour characteristic of high velocity crushing of cellular materials can in practice be modelled using shock wave analysis in continuum solids [6–8].

Various analytical approaches are employed in 1-D modelling of the shock like behaviour of cellular materials under in-plane high-velocity loading; these include: continuum models (underpinned by conservations laws and Rankine–Hugoniot jump relations) [9–15]; shock-wave models (based solely on Rankine–Hugoniot jump relations) [6,9,16] and; spring-mass models [6,17,18]. The focus of the present work is directed towards the development of a framework for continuum-based analytical models and so hereafter only the continuum-based models are reviewed and discussed.

All continuum-based analytical models found in the literature, rely on a common approach which is the use of conservation laws (for mass, linear momentum and energy) and the application of Rankine–Hugoniot jump relations. The principal difference is the type and nature of the material models or constitutive relations [6–8,10,11,15,19,20]. A particular example is the continuum-based analytical model proposed by Reid and Peng [21] for the modelling of uniaxial compression of wood specimens, where a simplified rigid-perfectly-plastic-locking (R-P-P-L) material model is applied.

The focus during the past two decades has been on the modification and development of more accurate material models and

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constitutive relations without significant change in the analytical approach for the analysis of continuum shock physics; this relies on the employment of differential forms of conservation laws for the continuous parts of the body and use of Rankine–Hugoniot jump conditions at the place of a discontinuity or shock. Examples include the work of Tan et al. [20] where a thermo-mechanical model is applied to high-velocity crushing of aluminium foams with the R-P-P-L idealisation for the material behaviour; good agreement with the experimental data is reported. Similarly Lopatnikov et al. [15,19] considered the effect of elasticity and applied the elastic-plastic-locking (E-P-L) and elastic-perfectly-plastic-locking (E-P-P-L) material models to a cellular Taylor bar and involving a rigid mass striker. Two recent continuum-based models presented by Zheng et al. [7,8] account for high velocity impact along with a transition-mode model for moderate-velocity crushing of cellular bars. These models utilise a rate independent rigid-plastic-hardening constitutive model. Harrigan et al. [6] presented a detailed review of the 1-D cellular bar crushing analytical models covering continuum-based, shock-wave and spring-mass schemes. They also studied the influence of elastic property in the crushing behaviour of aluminium foams through elastic, perfectly-plastic, hardening (E-P-P-H) material models for a target cellular bar.

The continuum-based models presented in the literature are all founded on differential representations of the governing conservation equations as opposed to integral transport forms (denoted transport equations here) and employ the concept of the mass-element which is usually treated in a Lagrangian frame. The state of the mass-element is required to be considered at the beginning and at the end of a finite-time step (before and after experiencing the shock). This approach is necessary to overcome a particular deficiency with the differential approach; it cannot directly be applied to a discontinuity. Integration however can be applied directly which is a particular advantage that transport equations have, which is exploited in this work. It is worth highlighting here that conservation laws reference flows of physical quantities through a control volume and consequently are well described through integration, which in the absence of any discontinuity (i.e. material and geometrical) give rise to differential representations.

The present technique is based on three main features which are:

1. The governing conservation equations are presented in the form of integral transport equations applicable to a moving/deforming control volume.
2. A control volume transporting within another moving control volume is applied to capture shock physics. One moving-control-volume (MCV) tracks the shock and the other MCV is selected judiciously to enclose part of, or the whole continuum body.
3. Transport equation representations are applied invoking both mechanics and thermodynamics axioms; these reveal consistent results that conserve momentum and energy simultaneously.

The purpose of the presented work is threefold:

1. To establish a mathematical formulation for the analysis of deforming moving domains, particularly pertinent to discontinuous physics. The formulation links for the first time control volume movement to both transport and partial differential formulations.
2. To show how the new formulation can be readily applied to shock problems to reveal precise analytical solutions to seen and unseen problems.
3. To illustrate the ability of the new methodology to provide insight into an unresolved issue with momentum and energy solutions.

The efficacy of the approach is shown through three case studies which are respectively: Taylor cellular bar; mass impacting on the cellular bar and; the crushing of the cellular target with compacted material satisfying the Bingham plasticity material model [24,25]. Also inconsistency between momentum and energy methods identified by Harrigan et al. [6] is discussed in detail for the second case study. It is demonstrated that the problem is due to the erroneous application of the energy equation. The third case study represents an advance on existing analytical models for the deformation of high-velocity crushing of cellular bars.

2. Mathematical concepts underpinning the transport approach

The balance laws in continuum mechanics are derived by considering the axioms of mass conservation, conservation of linear and angular momentums and the laws of thermodynamics. Unlike Newtonian mechanics which is a particle-based approach and yields to differential governing equations, continuum mechanics is based on Euler's axioms which describe the continua as a continuum collection of particles which leads to integral governing equations. Hence the original form of the conservation laws for continua is written in the form of integral equations. Conservation laws are applicable to a moving control volume whose motion can be described by means of a velocity field $\underline{v}^*$. An interesting feature neglected in the literature is the possibility that different velocity fields can be used to the advantage of analytical analysis. In this work two velocity fields are utilised: $\underline{v}^*$ is selected to provide for general movement of a control volume (would move a mesh in any numerical analysis) and $\underline{v}^+$ is chosen to match the external boundary movement as described by $\underline{v}^*$ but additionally accounts for shock-wave movement. A control-volume travelling with a shock has its motion described by the velocity field $\underline{v}^+$.

2.1. Conservation governing equations

Consider the continuum body Ω^M depicted in Fig. 1 which moves with material velocity $\underline{v}$. Its initial state at $t = t_0$ can be defined to be the material reference system Ω^{M_0} and material points in the reference space are identified by coordinates $\mathbf{X}$. It is common practice in a Lagrangian frame to describe the movement of Ω^M using the identity $\underline{v} = D\boldsymbol{x}/Dt = \partial\boldsymbol{x}/\partial t|_{\mathbf{X}}$, which provides a first-order system for the solution of spatial coordinates $\boldsymbol{x}$. A unique solution $\boldsymbol{x}(\mathbf{X},t)$ is guaranteed by Frobenius's theorem [26] but this relies on $\underline{v}$

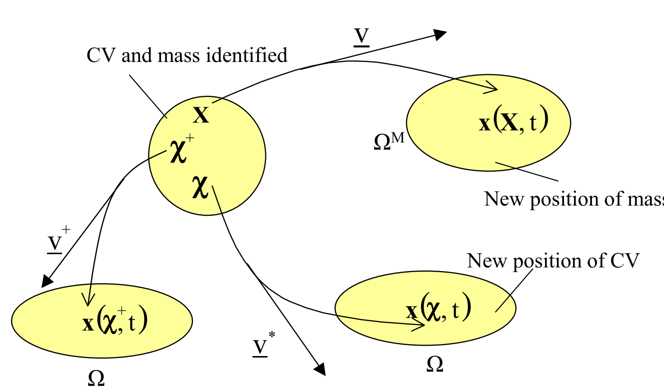

Fig. 1. Description of reference domains and their associated velocities.