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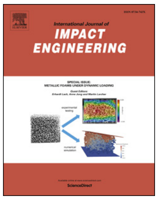

Non-physical finite element modelling of high speed normal crushing of cellular materials

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ABSTRACT

This work presents a new FE based methodology for numerical simulation of shock like behaviour in high speed crushing of metallic cellular materials. This recently developed numerical technique is called the non-physical finite element method. The approach is based on integral transport forms of the governing conservation laws and the concept of non-physical variable. With the non-physical variable concept each conservation law gives rise to a non-physical field. Unfortunately this results in a doubling of the number of transport equations and the number of field variables. However, non-physical variables can be shown to possess limiting continuity at any discontinuity in the physical fields. Another feature of the formulation is the presence of a non-physical source term at a discontinuity whose strength is related to the magnitude of the discontinuity in the physical field. One benefit of the approach is the precise annihilation of discontinuous behaviour from the governing finite element equations. Hence, classical continuous finite element approximations can be used with high accuracy to solve the resulting system of equations. The methodology is demonstrated through application to three different models of 1-D in-plane high-velocity impact crushing of a cellular Taylor bar, where numerical results are found to be in excellent correspondence to predictions from analytical models. The present work belongs to the category of finite element based shock capturing techniques.

1. Introduction

The reinforcement of structures and their shock resistance under high rate loading is an active research area in experimental, theoretical and numerical engineering [1]. Cellular materials in general and metallic cellular materials in particular are well known energy absorbers. They are widely used in advanced structural components due to their light weight or low specific weight and high energy absorbing capacitance and high impact resistance [2–4]. The metallic cellular materials (i.e. honeycomb and foams) are usually used as core in sandwich type or filler in other types of impact energy absorber structures such as shock mitigators and blast protectors [5–8].

In the case of unidirectional in-plane impact crushing of honeycomb type metallic cellular materials, with regular and irregular cellular structures, different deformation mechanisms can occur, which are denoted by X, V and I failure mode patterns [9]. The deformation patterns X and V prevail for low velocity impact and the associated distribution of random shear bands depends on the material and structural properties of the honeycomb [10]. For any metallic honeycomb with specific material and structural properties, there is a critical impact velocity, above which results in a narrow and localised I-pattern collapse band [10]. In general therefore, moderate velocity impact crushing of honeycombs results in random collapse bands but for relatively high velocities a localised phenomenon occurs with layer-by-layer collapse bands [11]. In high velocity impact crushing of cellular bars, the localised deformation propagates in a similar fashion as a plastic wave, moving from one end of the bar to another [7]. The very low thickness of the localised narrow band (i.e. I-pattern) and the marked difference between physical fields (i.e. density, velocity, stress) of the crushed and uncrushed part of the cellular bar is reminiscent of the propagation of a plastic-shock wave. This shock like behaviour only happens for the crushing of the cellular materials with impact velocities above a critical impact velocity [7,10,11]. The scope of the present work is to provide a finite element method (FEM) based shock capturing technique for the numerical study of shock-like behaviour of metallic cellular materials under high speed impact crushing.

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The available finite element (FE) based techniques for the modelling of high velocity impact crushing of cellular bars reported in the literature falls into two main categories, i.e. continuum solid formulations and multi-scale/cellular models [11–18].

The continuum solid FE approaches represent the porous cellular material with a non-porous continuous structure and appropriate constitutive relationship, and apply the classical FEM. The deformation mechanisms are incorporated by means of experimentally validated constitutive relations [14,18–24]. Although these models can provide acceptable results in comparison with experimental data, they lack a rigorous numerical foundation. The problem is their reliance on continuous classical FE to capture the discontinuous physics involved with the crushing of cellular materials. It is appreciated that it is not possible to capture shock or jump like behaviour using pure continuous Galerkin finite element discretisation.

The alternative simulation technique (multi-scale/cellular FEM) is underpinned by a sound theoretical foundation. The approach represents the porous structure of the metallic cellular material with regular/irregular macro-scale hexagonal cells in metallic honeycombs or micro/meso-scale cells in metallic foams [7,9–13,25–35]. Rather than defining deformation mechanism through constitutive relations it captures the deformation mechanism by modelling cell behaviour using beam and shell finite elements. Although this FE approach is theoretically well founded it suffers in that it is computationally expensive especially for practical cellular structures [15,16,27,32,36,37]. Also it suffers from mesh sensitivity and convergence difficulties in the case of metallic foams with irregular micro/meso-scale structures [15,16].

None of the mentioned numerical approaches are placed in the category of the FE based shock capturing techniques. Each makes use of continuous Galerkin finite element method (CGFEM) based commercial software although applied over different geometrical scales. The absence of a robust FE based technique with shock capturing features but founded on classical FE has lead to the development of non-physical finite element method (NPFEM). This approach is shown to be able to capture the shock-like response of cellular materials undergoing high-velocity impact. The NPFEM models the cellular material as a continuous non-porous solid but at the same time is able to capture shocks as they propagate through continuous solid elements [38,39].

Shock modelling remains a computationally resource-intensive activity with traditional techniques unable to readily represent sharp localised behaviour without the deployment of significant resources in the form of high mesh densities, special elements, re-meshing, reduced time steps etc. [40]. Non-traditional methods not involving meshes do exist and are reported to be advantageous for shock modelling but presently these methods have no significant commercial exposure and the cost of bringing them to the marketplace is prohibitive [41].

Most of the current mesh-based macro-scale techniques for modelling high-rate phenomena are founded on difference equations such as finite differences or finite volume methods [42]. Finite element techniques such as the discontinuous Galerkin finite element method (DGFEM) or the extended FEM (XFEM) have the capability of modelling material discontinuities and shock problems but they suffer from issues such as singularity, poor conditioning, instability and significant extra computational costs [43,44]. The interpolation functions employed in these methods do not belong to the standard Sobolev space H^1 underpinning classical finite elements [43,45], hence added complexity and cost is involved in the incorporation of these approaches in existing commercial software.

Although it is generally accepted that there is no feasible way of incorporating an inter-element shock in the space H^1 a recent innovation of NPFEM has allowed precisely that; which opens up the possibility for efficient shock modelling in standard finite element codes [38,39].

The non-physical finite element method (NPFEM) is principally founded on the concept of the non-physical variables and their relation to the physical fields through the equivalent transport equations [39]. Equivalent transport equations written in terms of non-physical variables describe the behaviour of non-physical variables and are exact replacements for the original governing equations. The governing equations are presented in the form of transport equations because these encompass discontinuous behaviour without approximation and are applicable to moving control volumes. The non-physical variables are a completely novel mathematical concept involving the defining of non-physical variables for each corresponding physical variable (e.g. density, enthalpy, entropy, velocity etc.). With a precise definition, a non-physical variable can be shown to have limiting continuity at a shock but additionally generates a source term which tracks with the shock front. In other words, the non-physical method replaces the discontinuous physical fields or variables with a limiting-continuous non-physical variable. Limiting continuity means that the non-physical variables are continuous at ever point except at the place of the shock where they pose a source-like behaviour. A schematic 1-D representation of how NPFEM replaces the discontinuous physical field ψ by a limiting continuous non-physical variable $\widehat{\psi}$ and non-physical source $\widehat{\psi}'$ is illustrated in Fig. 1. It is clear that the non-physical variable can be discretised using continuous Galerkin shape functions.

Another aspect of the NPFEM is the application of a moving control volume which tracks the shock wave. Traditionally, a control volume can be selected to be moving, stationary, mass tracking, part of, or enclosing the whole system domain. To model a discontinuity, a moving control volume (MCV), which generally, is in another moving CV, is required. The first MCV includes the shock and the second MCV encloses part of, or the whole system domain. A feature of the non-physical approach is that non-physical variables are affected by the manner in which an MCV is transported. This raises problems for the analysis in a situation which has an MCV within another MCV. However, by means of bespoke transport equations (TEs) the analysis can be performed and the approach is denoted here by an MCV–MCV analysis [39].

This limiting continuity property possessed by non-physical variables can be shown through an MCV–MCV analysis to facilitate the precise annihilation of the discontinuity from the NPFEM system of equations. This means that interpolants can be selected from the standard H^1 space and the Galerkin-weighted system of NPFEM formulation can be discretised using continuous Galerkin shape functions and solved in a traditional manner.

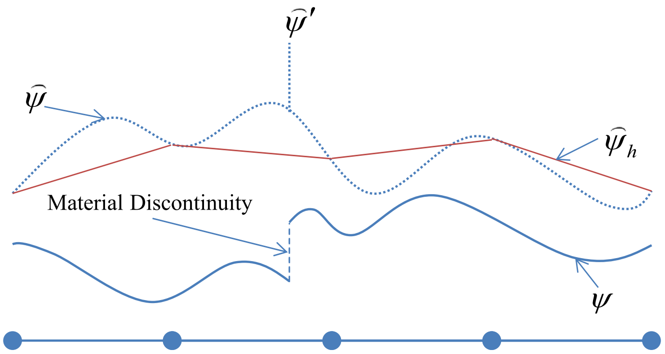

Fig. 1. Schematic 1-D descritisation of a discontinuous function in NPFEM.