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The dynamic stability of a rotating pre-twisted asymmetric cross-section Timoshenko beam subjected to lateral parametric excitation

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Abstract

A finite element model is presented to study the dynamic stability of a pre-twisted Timoshenko beam having asymmetric aerofoil cross-section subjected to lateral parametric excitation. Solutions referred to as combination resonance are investigated. The effects of shear deformation and rotary inertia are included in the analysis. The effects of coupling due to centre of flexure distance from the centroid and the shear coefficient on the stability are considered. The change in these effects due to pre-twist angle is investigated. It is concluded from the results that the pre-twist angle has an influence on the effects of coupling and the shear coefficient.
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Keywords: Dynamic stability; Timoshenko beam; Lateral; Rotating; Pre-twist; Asymmetric

1. Introduction

Various engineering components such as compressor, turbine and helicopter rotor blades are pre-twisted beams and the dynamic stability of rotating pre-twisted beams subjected to lateral excitation is of considerable importance at the design stage. In general the modes of vibration of pre-twisted asymmetric aerofoil cross-section blades are of the coupled bending–bending–torsion types. For a beam under lateral parametric excitation, the governing equations reduce to systems of Mathieu equations coupled mainly by symmetric, off-diagonal terms [19]. Dominant instability regions for such Mathieu equations occur at forcing frequencies equal to sums of the natural frequencies in bending and torsion. Such solutions are referred to as combination resonance [19]. The effects of shear deformation and rotary inertia on the stability and vibration analysis of non-slender beams, where the length to depth ratios are small, are taken into consideration.

Several investigations about the vibration characteristics of asymmetrical blades under rotating and non-rotating conditions have been carried out. Carnegie [1] developed a set of equations defining the dynamic motion of a pre-twisted aerofoil blading and investigated the effect of pre-twist on the frequencies of vibration using the Rayleigh–-Ritz method. Carnegie and Dawson [2,3] studied the vibration characteristics of straight and pre-twisted aerofoil blades by transforming the equations of motion to a set of simultaneous first-order differential equations and by a step-by-step integration using the Runge–Kutta procedure. Carnegie [4] derived the equations of motion of a rotating aerofoil cross-section blading allowing for pre-twist and stagger angle. Rao and Carnegie [5] applied the Galerkin procedure to determine the natural frequencies of a straight cantilever blade with an asymmetrical aerofoil cross-section executing coupled bending torsion vibrations. Carnegie and Thomas [6] investigated the effects of shear deformation and rotary inertia on the lateral frequencies of flexural vibration of pre-twisted and non-pre-twisted uniform and tapered cantilever beams. Rao and Banerjee [7] developed a polynomial frequency equation method to determine the natural frequencies of a cantilever blade with an asymmetric cross-section mounted on a rotating disc. Thomas and Sabuncu [8] presented a finite element model for the dynamic analysis of an asymmetric, cross-section blade. Fu [9] derived basic equations for a comprehensive computer analysis for a lumped parameter system.

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Nomenclature

A	area of blade cross-section
C	torsional stiffness
c_f	centre of flexure
c_g	centroid
E	modulus of elasticity
G	modulus of rigidity
d_X, d_Y	coordinates of centre of flexure with respect to X and Y
d_x, d_y	coordinates of centre of flexure with respect to x and y
$\bar{d}_x$, $\bar{d}_y$	non-dimensional parameters corresponding to d_x and d_y
k	shear coefficient
$\bar{k}$	non-dimensional shear parameter $(kG/E)^{1/2}$
I_{XX}, I_{YY}	area moments of inertia of cross-section about XX and YY
I_{xx}, I_{yy}	area moments of inertia of cross-section about x_1x_1 and y_1y_1
I_{xy}	product moment of inertia of cross-section about x_1x_1 and y_1y_1
J_z	polar moment of area of cross-section
I_{cf}	moment of inertia about the centre of flexure, $I_{cf} = J_z + (d_x^2 + d_y^2)A$
ϕ_s	stagger angle
α_p	angle between coordinate axes (x_1x_1, y_1y_1) and principal axes (XX, YY) angle of pre-twist
α_i	pre-twist angles of each beam element at their starting points
α_b	pre-twist angle along the beam element
L	blade length
l	element length
R	disc radius
T^e	kinetic energy of the beam element
U^e	potential energy of the beam element
U_R^e	potential energy of the beam element due to rotation
U_s^e	strain energy of the beam element
V_p^e	potential energy of the beam element due to external force
T	kinetic energy of the beam
U	potential energy of the beam
U_R	potential energy of the beam due to rotation
U_s	strain energy of the beam
V_p	potential energy of the beam due to external force
XX, YY	principal axes through centroid of blade cross-section
xx, yy	coordinate axes through centre of flexure of blade cross-section
x_1x_1, y_1y_1	coordinate axes through centroid of blade cross-section
$\xi\xi$	coordinate axis through centroid of blade cross-section tangential to the plane of disk
$\delta\delta$	coordinate axis through centroid of blade cross-section normal to the plane of disk
u	displacement of centre of flexure in x direction
u_1	displacement of centroid in x direction
v	displacement of centre of flexure in y direction
v_1	displacement of centroid in y direction
z	coordinate distance measured along the length of blade from root
ϕ_x	bending slope in yz plane
ϕ_y	bending slope in xz plane
θ	torsional displacement
ω	rotational speed
ψ_x	non-dimensional u deflection
ψ_y	non-dimensional v deflection
ψ_{x_1}	non-dimensional coordinate corresponding to u_1
ψ_{y_1}	non-dimensional coordinate corresponding to v_1
η	non-dimensional coordinate corresponding to z
M	mass matrix
K_e	elastic stiffness matrix
K_g	geometric stiffness matrix
Λ	diagonal eigenvalue matrix
Ψ	normalized geometric stiffness matrix
Ω	disturbing frequency
λ	frequency parameter of external loading
Ω_j	jth natural frequency
Ω_{jk}	$\Omega_j + \Omega_k$
λ_j	jth natural frequency parameter
λ_{jk}	$\lambda_j + \lambda_k$
G_{jk}	width parameter of unstable region around λ_{jk}
ρ	density
n	number of elements
p	circular natural frequency
p_0	fundamental natural frequency without pre-twist
ρ	density
n	number of elements
Γ	angle of loading

Karadag [10] investigated the dynamic characteristics of rotating and non-rotating practical bladed disks by taking blade shear centre effects into account. Subrahmanyam et al. [11] presented natural frequencies and modal shapes of the first five modes of vibration for a rotating blade of asymmetrical aerofoil cross-section with allowance for shear deflection and rotary inertia. Steinman et al. [12] described both vibration and buckling behaviour for a

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