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Exact static analysis of partially composite beams and beam-columns

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Abstract

The ordinary differential equations and general solutions for the deflection and internal actions and, especially, the pertaining consistent boundary conditions for partially composite Euler–Bernoulli beams and beam-columns are presented. Static loading conditions, including transverse and axial loading and first- and second-order analyses are considered. The theoretical procedure is applicable to general loading and boundary conditions for uniform composite beams and beam-columns with interlayer slip. Further, the exact closed form characteristic equations and their associated exact buckling length coefficients for composite columns with interlayer slip are derived for the four Euler boundary conditions. It is shown that these coefficients are the same as those for ordinary fully composite (solid) columns, except for the Euler clamped-pinned case. For the clamped-pinned case, the difference between the exact buckling length coefficient and the corresponding value for solid columns is less than 1.8% depending on the so-called composite action parameter and relative bending stiffness parameter. Correspondingly, the maximum deviation between the exact and approximate buckling load is at most 2.5%. These small differences can in most practical cases be neglected. Also, the maximum theoretical range for the relative bending stiffness for partially composite beams and beam-columns is derived. An effective bending stiffness, valuable in the determination of the critical buckling load for partially composite members, is derived. This effective bending stiffness is also suitable for analysing approximate deflections and internal actions or stresses in composite beams with flexible shear connection. The beam-column analysis is applied to a specific case. The difference in the approaches to the first- and second-order analysis is illustrated and the results clearly show the magnification in the actions and displacements due to the second-order effect. The magnification of the internal axial forces is different from magnifications obtained for the other internal actions, since only that portion of an internal axial force that is induced by bending is magnified by the second-order effect.

Keywords: Composite beams/columns; Partial interaction; Interlayer slip; Elastic buckling; Buckling length coefficients; Variational methods

1. Introduction

Composite structures of different materials are frequently used in building, bridge and shelter construction. In building construction, composite structures are often used as floor and wall elements, e.g. timber–concrete elements composed of thin concrete plates attached to wood studs by means of shear connectors. These structures can be subjected to different kinds of static and dynamic transverse loadings, and when used as wall panels they are also subjected to axial loads. This paper deals with the static applications and a companion paper considers the corresponding dynamic applications, [1].

The behaviour of these members is quite complex because the shear connectors generally permit the development of only partial composite action between the individual components of the member, and their analysis requires the consideration of the interlayer slip between the subcomponents.

The fundamental equations of the theory for one-dimensional, linear elastic, partial composite action for beams and columns subjected to static loads was first developed by Stüssi [2], Switzerland, Granholm [3], Sweden, Newmark et al. [4], USA, and Pleshkov [5], Russia, independently. The theory has been applied by many analysts in the analysis of composite beams and columns with interlayer slip, e.g. Goodman [6], Amana and

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Booth [7], concerning static loads, and Henghold [8], concerning dynamic loads, to mention a few of the early works. One of the authors applied the partial composite action theory to beams subjected to static loads, Girhammar [9], and to dynamic loads, Girhammar [10], and to beam-columns subjected to static loads, Girhammar [11]. Earlier analysis of composite beam-columns conducted by Girhammar and Gopu [12] dealt with only one particular axial loading case; they extended and generalized their theory later, Girhammar and Gopu [13]; and Girhammar and Pan [14] analysed composite members subjected to dynamic loads; see also Pan [15]. Recently, Ranzi et al. [16] presented general analytical solutions for statically loaded composite beams with partial interaction and Faella et al. [17] an "exact" finite element model based on "exact" analytical expression of the stiffness matrix. Ranzi and Bradford [18] and Jurkiewiez et al. [19] presented analytical solutions and a numerical model, respectively, for time-dependent behaviour of partially composite beams. Different kinds of numerical and finite element formulations for the analysis of composite beams with interlayer slip have been suggested, e.g. by Ayoub [20,21], Ranzi et al. [22] and Cas et al. [23]. Gara et al. [24] developed a finite element model for composite beams with both longitudinal slip and vertical uplift of the interface connection. Dall'Asta [25] developed a three-dimensional theory for composite beams with shear weak connection dealing with combinations of bending in the symmetry plane, and torsion and transverse bending in the plane parallel to the shear connector interface.

The classical approach to the solution of structural and solid mechanics problems, like the one mentioned above, is to formulate the governing differential equation and obtain the analytical solution. The corresponding boundary conditions are stated so as to satisfy the apparent geometrical and mechanical conditions at the boundaries. The stated set of boundary conditions, which reflects the investigators view of the conditions at the boundary, need not only be mechanically correct, the resulting formulation needs also be mathematically well posed, i.e. mathematically consistent. A number of such boundary value problems in structural mechanics, whose intuitive formulation led to incorrect boundary and matching conditions, have been published, see e.g. Kerr [26]. Examples of attention to this critical issue of boundary conditions derived consistently with the energy functional in other related fields are the works of Askes and Metrikine [27] and Landis [28]. Thus, for some problems it is not a straight forward procedure to mathematically describe the geometrical and boundary conditions in a consistent way.

For composite members with interlayer slip, boundary conditions chosen by inspection of what have appeared to be the physical conditions of the body have later been proved to be wrong. For example, for a built-in end of a partially composite member, the conditions of zero deflection and zero angular rotation, but non-constant slip were assumed, [29]. It will be proved in this paper that a constant slip value (for example, zero) is a necessary and consistent boundary condition in order to render a well-posed formulation.

The strength of variational methods is that the particular function that gives the minimum value of the potential of the problem is the solution to the governing differential equation. The procedure gives at the same time all the admissible boundary conditions. From these boundary conditions a well posed formulation, mechanically and mathematically, may be chosen. These methods are well established and widely used, see e.g. Washizu [30] and Oden and Reddy [31].

Girhammar and Gopu [13] developed in the classical way the governing sixth-order ordinary differential equation and its general solution for composite beam-columns with interlayer slip subjected to static loading, but did not discuss the proper boundary conditions. Also, they did not evaluate the exact buckling length coefficients or exact buckling loads as a function of the degree of composite action and the relative bending stiffness for the various Euler cases. They only proposed to use the buckling length coefficients valid for ordinary fully composite (solid) columns in the evaluation of approximate buckling loads for partially composite columns. A similar approximate procedure is adopted for buckling of sandwich columns, Plantema [32].

This paper is an extension and generalization of the theories and procedures given in Girhammar and Gopu [13]. The purpose of this paper is to derive, by using variational methods, the ordinary differential equations for the deflection and internal actions and all the pertaining admissible boundary conditions for partially composite Euler–Bernoulli beams and beam-columns, both of first and second order. In particular, the aim is to derive the exact closed form characteristic equations within the validity of the Euler–Bernoulli beam theory and the associate buckling length coefficients for partially composite beam-columns. These equations and coefficients are derived for the four different Euler cases as a function of the degree of composite action and the relative bending stiffness. The exact buckling length coefficients are also compared to the corresponding coefficients for fully composite (solid) beam-columns. In this paper, the coefficients for solid beam-columns are called approximate coefficients when they are applied also to partially composite beam-columns. An evaluation of the accuracy of using these approximate buckling length coefficients in the determination of the fundamental buckling loads for partial composite columns is also made. The theory is applied to simply supported beam-columns in order to illustrate the magnification in the internal actions and displacements due to the second-order effects. An example of derivation of exact characteristic equations in the dynamics of vibrations in another related field is the work of Hashemi and Arsanjami [33]. For a corresponding analysis with respect to deriving the governing equations, consistent boundary conditions and exact characteristic