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Large deflection response of an elastic, perfectly plastic cantilever beam subjected to a step loading

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A B S T R A C T

This paper studies the large deflection response of an elastic, perfectly plastic cantilever beam subjected to a step load using finite difference method. A computational stability requirement is proposed to determine the relationship between the time-step and the element size. It is demonstrated that the numerical solution is convergent if the stability requirement can be satisfied. The deformation mechanism of the cantilever beam is studied based on the instantaneous distributions of bending moment and curvature during the response of the beam. It is found that the deformation mechanism depends on the magnitude of the step load. When the step load is moderate, only a single stationary plastic bending hinge is formed at the root of cantilever beam during the response. For intensive loading magnitude, stationary plastic bending hinges at both interior and root positions are formed with the latter as the predominant deformation mode during the response of the cantilever beam, which is supported by the rigid, perfectly plastic analysis. The limitations of the classical rigid, perfectly plastic cantilever beam model are discussed.

1. Introduction

It is generally accepted that, when the input energy to a structure is much larger than the maximum elastic energy that can be stored in the structure, the dynamic structural response can be satisfactorily predicted by a rigid-plastic model [13], in which the governing equations of the structure can be greatly simplified by neglecting elastic effects on the plastic deformation. For example, in structural responses with small deflection, all bending deformations in a rigid, perfectly plastic beam, plate or shell are localized into discrete plastic hinges and the corresponding governing equations may be solved analytically [5]. However, it is difficult to define an accurate valid range for the application of a rigid-plastic model. As a result, rigid-plastic simplification has been questioned when the neglected elastic deformation may play significant roles in the plastic response of the structure [10,14].

Rigid-plastic dynamic response of a cantilever beam subjected to a step load at its tip has been studied as a classical problem in the dynamic plastic response of structures [12]. Based on two basic assumptions, i.e. small deflection and rigid, perfectly plastic idealization, various deformation mechanisms may be initiated in a cantilever beam according to the magnitude of the step load [12], i.e.

$$\begin{aligned} &0 < F \le F_c \quad \text{when the whole beam retains rigid} \\ &F_c < F \le 3F_c \quad \text{when the stationary plastic bending hinge is formed at the root of beam} \\ &3F_c < F < \infty \quad \text{when the stationary plastic hinge is formed at the interior position of beam} \end{aligned} \qquad (1a\text{–}c)$$

where F_c denotes the plastic collapse load under static loading ($F_c = M_p/L$ where L is the length of the cantilever beam; M_p is the fully plastic bending moment of the cross-section of the beam. For a rectangular cross-section described in Fig. 1, $M_p = \sigma_s bh^2/4$ where σ_s is the yield stress of the beam material). This simplified model has been extended to more complicated cases, such as cantilever beams with variable and stepped cross-sections [4,22] and cantilever sandwich beams [21].

However, it has been noted that existing analytical models for cantilever beams, in which the governing equations are based on the original configuration of the beam, are valid only for small deflections. In practice, the configuration of the beam can be changed considerably under intensive impact loading, and therefore, the consideration of the large deflection in governing

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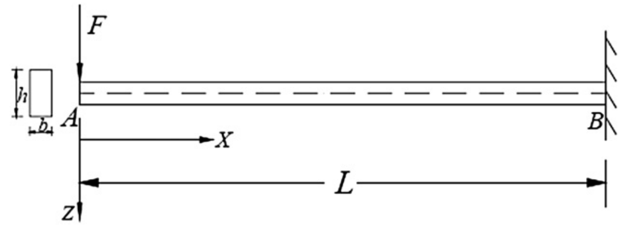

Fig. 1. A uniform cantilever beam subjected to a transverse load at its tip.

equations may be necessary in some circumstances. Based on authors' best knowledge, no publications have considered large deflection effects in the rigid-plastic dynamic response of a cantilever beam subjected to step load.

On the other hand, when a rigid-plastic model is employed, the idealization is based on the neglecting of elastic deformations. The elastic effect on the bending response of a rigid, perfectly plastic cantilever beam has been studied by introducing a rotational spring at the root of the cantilever beam [16,17,20], in which it was found that the interactions between the root spring and the traveling plastic hinge influence the rigid-plastic response mode in the cantilever beam. However, a root spring cannot fully represent the interactions between elastic and plastic flexural waves in the beam. A further study on this issue was presented in [21] where an elastic-plastic constitutive relation was used to study the interaction between reflected elastic flexural waves and a plastic bending hinge in the dynamic response of pulse-loaded beams. This study focused on the early-time response of the beam when the formation of the plastic bending hinge is an outcome of the interactions between elastic and plastic flexural waves. Influence of elastic shear deformation on the plastic shear hinge of a fully clamped beam subjected to blast loading is demonstrated in [8]. These studies are limited to their valid response stages when influence of large deflection on the beam response can be neglected.

In a study of pipe-whipping phenomenon [11], the large deflection effect is considered in the established dynamic equations for the elastic-plastic response of a cantilever beam but the rotary inertia effect is neglected. It was found in [21] that the rotary inertia plays an important role for the early flexural response of the beam. Therefore, it is necessary to develop a general approach to take elasticity, rotary inertia and large deflection into consideration in the whole dynamic response period of a beam when it is subjected to the whole range of dynamic load.

In this paper, an elastic, perfectly plastic cantilever beam subjected to a step load at the tip of the beam, which has been investigated intensively as a typical dynamic plastic response problem, is employed to address above concerns. Following assumptions are adopted in the present analysis, i.e. (a) Euler-Bernoulli beam theory with the consideration of rotary inertia is employed, and (b) the material is elastic, perfectly plastic and rate-independent and the influence of shear stress on yield is not considered. Governing equations based on large deflection are established and then solved numerically using finite difference scheme. A requirement defined by a relation between time incremental step and the element size in finite different method is recommended for the computational stability of the governing partial differential equations for the beam in large deflection. The convergence of the solution is verified by comparing present solutions with ABAQUS predictions. Based on the validated numerical model, the influence of finite deflection on the response modes of the cantilever beam subjected to a step loading at its tip is studied.

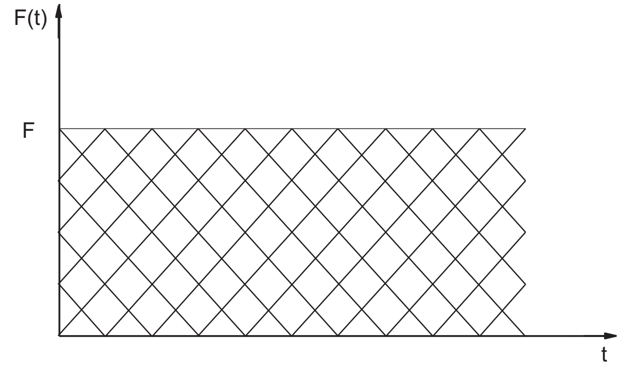

Fig. 2. Step load originating at $t = 0$.

2. Governing equations and numerical technique

2.1. Governing equations

For simplicity, a rectangular cross-sectional cantilever beam with length L, width b and depth h is considered (Fig. 1). However, the governing equations developed in this section can be easily extended to cantilever beams with non-rectangular cross-sections. The density of material, yield stress and Young's modulus are ρ, σ_s and E, respectively. A step load, $F(t)$, as shown in Fig. 2, is applied at the tip of the cantilever.

The governing equations are initially derived for a general loading case, i.e. the beam element is loaded by distributed external forces with components $F_X(s,t)$ and $F_Z(s,t)$ per unit length within its plane of symmetry. A typical infinitesimal element of length ds is shown in Fig. 3. The center of a typical cross-section has displacement components u and w in X and Z directions, respectively. The internal generalized forces acting on the element are the shear force (Q), the axial force (N) and the bending moment (M). The equations of motion of the cantilever beam with considering large deflection, but neglecting rotary inertia were presented in [3]. Rotary inertia may have significant influence on the distribution of instantaneous bending moment along the beam at early stage when the elastic bending wave is dominant [21]. With the consideration of the rotary inertia and large deflection, the governing equations of the cantilever beam are,

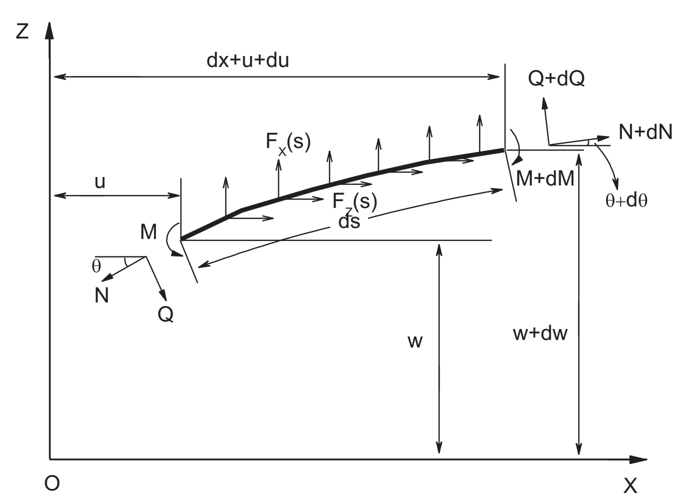

Fig. 3. A typical element of beam.