# Stabilized SPH-based simulations of impact dynamics using acceleration-corrected artificial viscosity

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ABSTRACT

Artificial viscosity in SPH-based computations of impact dynamics is a numerical artifice that helps stabilize spurious oscillations near the shock fronts and requires certain user-defined parameters. Improper choice of these parameters may lead to spurious entropy generation within the discretized system and make it over-dissipative. This is of particular concern in impact mechanics problems wherein the transient structural response may depend sensitively on the transfer of momentum and kinetic energy due to impact. In order to address this difficulty, an acceleration correction algorithm was proposed in Shaw and Reid ("Heuristic acceleration correction algorithm for use in SPH computations in impact mechanics", Comput. Methods Appl. Mech. Engrg., 198, 3962–3974) and further rationalized in Shaw et al. (An Optimally Corrected Form of Acceleration Correction Algorithm within SPH-based Simulations of Solid Mechanics, submitted to Comput. Methods Appl. Mech. Engrg). It was shown that the acceleration correction algorithm removes spurious high frequency oscillations in the computed response whilst retaining the stabilizing characteristics of the artificial viscosity in the presence of shocks and layers with sharp gradients. In this paper, we aim at gathering further insights into the acceleration correction algorithm by further exploring its application to problems related to impact dynamics. The numerical evidence in this work thus establishes that, together with the acceleration correction algorithm, SPH can be used as an accurate and efficient tool in dynamic, inelastic structural mechanics.

## 1. Introduction

Numerical modeling of dynamic response of structures under ballistic impacts has always challenged the ingenuity of researchers mainly owing to the presence of layers with sharp gradients. Element-based methods, employing the weak form of the governing equations, have a long and successful history in many branches of engineering and science. However, the requirements of continuity of the field variables within an element, finiteness of the element domain and bijections between undeformed and deformed elements limit their applications to impact dynamic problems that often involve very large or discontinuous deformations, material fragmentation and separation caused by cracks. Such limitations have been a motivation for particle-based methods (i.e., those not relying on element-based discretization), of which the smoothed particle hydrodynamics (SPH) is by far the most prominent one [1,7,8]. The domain discretization in SPH is through a set of particles, which interact with each other through a kernel function such that at every particle location the conservation equations are satisfied. Over the last decade, several attempts [3,9,16] have been made to explore the potential of SPH in problems related to impact mechanics.

In the standard SPH computation, in order to capture the sudden jump in the physical quantity and promote the numerical stability, an artificial viscous term is added in the linear momentum balance equation. The basic physical attribute corresponding to artificial viscosity is numerical dissipation, i.e. it converts kinetic energy to internal energy. The strength of the artificial viscosity can be controlled by certain user-defined parameters, called the artificial viscosity parameters. If artificial viscosity yields numerically stable yet minimally dissipated solutions, its strength is said to be "optimum". The parameter values that optimize the strength of artificial viscosity depend on such factors as particle distribution, intensity of the shock etc. and therefore are not known a-priori. Unfortunately there is no standard set of such parameters that work for a wide range of problems. The common practice is to choose some arbitrary values of these parameters irrespective of

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the computational data (particle distribution, kernel function, smoothing length etc.) and nature of the response. Improper choice of these parameters may lead to an excessive loss of kinetic energy, making the system over–dissipative and thus the predictions physically unreal [4,13,14,15]. By way of an attempt to overcome this limitation, an acceleration correction algorithm (AC) was proposed by Ref. [13] and later rationalized by Shaw and Reid (2010). The essence of the method is to assess the change in the acceleration due to the artificial viscosity term and then correct the computed acceleration by subtracting a convex approximation of the 'changed' acceleration. The strength of the correction term in the momentum balance equation was optimized by taking a spatially varying response-dependent support size of kernel function through which the correction term is computed. Through limited numerical experiments, it was demonstrated in [13,14] and Shaw et al. (2010) that the acceleration correction algorithm, while stabilizing the numerical solution without much unwanted dissipation, also bypasses the need any user specified viscosity parameters.

While the limited nature of numerical evidence, obtained so far with the acceleration correction algorithm, is suggestive of its broader applicability to a large class of problems in impact dynamics, a more conclusive proof of this observation can only be had through its further numerical exploration to which the present paper is mainly devoted. The superior performance of the SPH with acceleration correction algorithm vis-à-vis the SPH with standard artificial viscosity is thus discussed through several numerical examples. In the original formulation of the acceleration correction algorithm [13,14] and its subsequent development [15], the scheme was applied to the Monaghan type artificial viscosity as it was the most widely used form in SPH computations in solid mechanics. In this work, the acceleration correction algorithm is viewed as an add-on which can be usefully employed with any form of artificial viscosity. In order to numerically demonstrate this, the algorithm is applied to two other forms of artificial viscosity viz. tensor product artificial viscosity by Owen [12] and a combined form of bulk viscosity and Von Neumann-Richtmyer viscosity as used by Hernquist L and Katz [2].

The paper is organized as follows. In Section 2, the main steps involved in SPH with acceleration correction algorithm are briefly outlined. The undesirable effect of the artificial viscosity and the efficacy of the acceleration correction algorithm to remove a significant part of it, in the context of some classical elastic–-plastic problems in dynamic structural mechanics, are discussed in Section 3. Extension of the acceleration correction algorithm to other form of artificial viscosity is given in Section 4. Conclusions are then drawn in Section 5.

## 2. SPH with acceleration correction

The SPH algorithm, corrected for acceleration, is briefly outlined in this section. For a more comprehensive description, readers are referred to [6,7,13,14] Shaw et al. (2010).

### 2.1. Governing equations

The conservation equations for continuum mechanics are given by

$$\frac{d\rho}{dt} = -\rho \frac{\partial v^\beta}{\partial x^\beta}, \tag{1}$$

$$\frac{dv^\alpha}{dt} = -\frac{1}{\rho} \frac{\partial \sigma^{\alpha\beta}}{\partial x^\beta}, \tag{2}$$

$$\frac{de}{dt} = -\frac{\sigma^{\alpha\beta}}{\rho} \frac{\partial v^\alpha}{\partial x^\beta}, \tag{3}$$

$$\text{and } \frac{dx^\alpha}{dt} = v^\alpha, \tag{4}$$

where, for any material point, denotes its mass density and $\rho$ the specific internal energy. $v^\alpha$ and $\sigma^{\alpha\beta}$ are respectively elements of the velocity vector and Cauchy stress tensor (symmetric), $x^\alpha$ is the spatial coordinate, $d/dt$ is the time derivative taken in the moving Lagrangian frame and the superscripts $\alpha, \beta = 1,2,3$ are integer indices for the three spatial directions.

### 2.2. Constitutive model

The stress components in Eq. (3) may be written in terms of hydrostatic and deviatoric stresses as,

$$\sigma^{\alpha\beta} = P\delta^{\alpha\beta} - S^{\alpha\beta} \tag{5}$$

where, $P$ and $S^{\alpha\beta}$ are respectively the scalar pressure and components of the traceless symmetric deviatoric stress tensor. The pressure in Eq. (5) may be calculated through an equation of state (EOS). Presently, we adopt the following EOS wherein pressure is assumed to vary linearly with the compression ratio.

$$P(\rho) = K\left(\frac{\rho}{\rho_0} - 1\right) \tag{6}$$

The components of $S^{\alpha\beta}$ may be obtained from the Jaumann stress rate, given by,

$$\dot{S}^{\alpha\beta} = 2\mu\left(\dot{\varepsilon}^{\alpha\beta} - \frac{1}{3}\delta^{\alpha\beta}\dot{\varepsilon}^{\gamma\gamma}\right) + S^{\alpha\gamma}\dot{R}^{\beta\gamma} + S^{\alpha\beta}\dot{R}^{\alpha\gamma} \tag{7}$$

$\mu$ is the shear modulus and $\delta^{\alpha\beta}$ is the Kronecker delta. The components $\dot{\varepsilon}^{\alpha\beta}$ of the symmetric strain rate tensor and $\dot{R}^{\alpha\beta}$ of the skew symmetric spin tensor are given by

$$\dot{\varepsilon}^{\alpha\beta} = \frac{1}{2}\left(\frac{\partial v^\alpha}{\partial x^\beta} + \frac{\partial v^\beta}{\partial x^\alpha}\right) \tag{8}$$

$$\dot{R}^{\alpha\beta} = \frac{1}{2}\left(\frac{\partial v^\alpha}{\partial x^\beta} - \frac{\partial v^\beta}{\partial x^\alpha}\right). \tag{9}$$

#### 2.2.1. Plasticity and yield criterion

In this paper, whenever plasticity is taken into account, the material is assumed to be elastic-perfectly-plastic and the flow régime is determined by Von Mises yield criterion. At every time step, the second stress invariant, $J_2 = S^{\alpha\beta}S^{\alpha\beta}$, is checked and if $\sqrt{J_2}$ exceeds the yield stress $\sigma_y/\sqrt{3}$ (where $\sigma_y$ is the uniaxial yield stress), the individual stress components are returned to the yield surface using

$$S^{\alpha\beta} \rightarrow fS^{\alpha\beta} \tag{10}$$

$$\text{where } f = \min\left\{\frac{\sigma_y}{\sqrt{3S^{\alpha\beta}S^{\alpha\beta}}}, 1\right\}. \tag{11}$$

Effective plastic strain increment is given by,

$$\Delta\varepsilon_p^{\alpha\beta} = \frac{1-f}{3\mu}S^{\alpha\beta} \tag{12}$$