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The impact response of clamped sandwich beams with ordinary and hierarchical cellular cores

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ABSTRACT

The response of clamped sandwich beams subjected to impact loading is analyzed based on the works of Fleck & Deshpande (2004) [9] and Reid et al. (2010) [14]. This study differentiates itself from that of Fleck & Deshpande in that the "conservation of momentum" method instead of the "energy balance" method is adopted to model the "compaction stage" of the core upon impact loading. Finite element method (FEM) is used to validate the developed analytical model and good agreement between the analytical method and FEM results is observed. Obtained results also show that, compared to the Fleck & Deshpande model, the present model gives improved predictions of the maximum lateral deflection of the front face and the boundary of the two regions where the cellular core is totally compacted and partly compacted. The developed model is then applied to study the effects of core relative density and core thickness on the maximum impulsive momentum that the sandwich beam can sustain (impact resistance), and near-optimum design is identified for a regular hexagonal core sandwich beam with given mass. In addition, based on the present model, the performance of sandwich beams with self-similar hierarchical hexagonal honeycomb cores under impact loading is studied. It is shown that, for given relative density, the strength of the self-similar hierarchical hexagonal honeycomb decreases with the hierarchical order increasing. Therefore, both the energy absorbed per unit mass of the core during the compaction stage and the impact resistance of the sandwich beam decrease as the hierarchical order increases.

1. Introduction

Cellular structures are well known for their excellent mechanical features, such as light weight, high toughness, high impact tolerance, high specific strength and stiffness [1–3]. A large number of experiments showed that metal foams are highly impact tolerant because their stress almost stays constant during the plastic deformation phase, where the nominal strain generally ranges from 0.5% to 75% [4,5]. This feature enables a large amount of energy being dissipated in the form of plastic energy, and thus makes metal foams excellent materials for impact protection: for example, the cushion pads used on the soft landing devices of lunar rovers, the bounding box of the automobiles, etc.

The impact of cellular structures has been studied extensively by analytical analysis, experiment, and finite element method. There are three analytical approaches currently existing to model the compaction wave propagating through the cellular structures under impact, namely, the 'shock' wave model [4–8], the "energy balance" model [3,9,10], and the "mass-spring" model [11,12].

Reid and Peng [7] tested the uniaxial dynamic crushing of wood and were the first to use the shockwave model based on the "conservation of momentum" theorem to explain the experimental results, where a rigid-perfectly-plastic-locking (R-P-P-L) model of wood was employed for the sake of simplicity. Later, similar shockwave analyses were conducted while different material models for the cellular structures and/or different impact loading modes were adopted. Hanssen et al. [4] studied the deformation behavior of a foam bar subjected to a linearly decaying blast loading with the R-P-P-L material model, Lopatnikov et al. [5] used an elastic-perfectly-plastic-rigid (E-P-P-R) model to study the impact of a mass upon metal foams, Harrigan et al. [6] studied the crushing behavior of wood using the rigid-softening-hardening (R-S-H) model and the elastic-softening-hardening (E-S-H) models. All the studies mentioned above showed good agreement between analytical predictions and experimental and/or numerical results.

Besides the conservation of momentum approach mentioned above, Tan et al. carried out experimental studies on the dynamic compressive properties of metal foams in Ref. [13] and later in a companion paper [8], proposed a 'shock' wave model based on

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thermo-mechanical approach which leads to the same governing equations to the that derived from "momentum theorem" in Ref. [7].

While most literatures studied the dynamic crushing of foams fixed at the distal end and loaded by impact on the proximal end [4–8,13,14], Fleck and Deshpande [9] (referred to as FD in the following) studied the blast resistance of clamped sandwich beams which included the analysis of compaction of the cellular core with both the front face and rear face free. They employed the "energy balance" approach to model the compaction wave propagating through the cellular arrays. The impact response of the sandwich beams was divided into three stages [9]: Stage I is the fluid–structure interaction which results in a uniform velocity of the front face; during stage II (compaction stage), the front face compresses the core to crush, leading to equal velocities of the faces and core; the beam goes on absorbing energy through plastic bending and stretching and finally comes to rest during stage III (retardation stage by FD [9]). The results of the FD model [9] were compared with the earlier FEM analysis of Xue and Hutchison [15], where the blast resistance of clamped sandwich plates was studied focusing on finding the near-optimal sandwich configurations.

Generally, cellular materials attenuate the loads induced by impact or blast because the stress transmission is limited by the plateau stress during the dynamic crush of cell arrays before densification begins. However, stress enhancement during the process has been observed [16–20], which can't be explained by the 'shock' wave model. Li and Meng [11] studied the stress enhancement phenomenon using a mass-spring model and the conditions distinguishing stress enhancement and attenuation regions have been identified. Gao and Yu [12] also studied the dynamic response of a cellular chain subjected to pulse loadings using mass-spring model, where fewer parameters were adopted to describe the stress-strain curve of cellular materials.

Harrigan et al. [14] explicitly compared the 'shock' wave model, the energy balance model, and mass-spring model. The advantages and disadvantages of these models were discussed [14]. In addition, it was argued [14] that the assumption made in energy balance model that the energy absorbed during dynamic compaction equals the energy absorbed during quasi-static compression is questionable.

Hierarchical structures, wherein the ribs of the traditional cellular materials are themselves cellular, have been proposed as a potential way to improve the mechanical properties of cellular materials. For example, Bhat et al. [21] manufactured and tested second order sandwich panels and found that their compressive strength is approximately six times greater than that of first order sandwich panels with the same weight. Lakes [22] proposed a recursive formula for self-similar hierarchical structures and showed that the compressive strength of second order honeycomb structures can be 3–4 times higher than that of equal mass first order honeycomb structures. Fan et al. [23] analytically studied the mechanical properties of second order hierarchical cellular materials made up of sandwich walls and revealed enhancement of mechanical properties compared with first order cellular materials. However, little work has been done to explore the impact resistance of hierarchical structures.

In this paper, the problem studied by FD is modified with the compaction stage modeled by the 'shock' wave model instead of the energy balance model. Here, we study the dynamic compaction of the cellular core with both the front face and rear face free using 'shock' wave model while in most literatures, the focus is on the dynamic crushing of foams fixed at the distal end and loaded by impact on the proximal end [4–8,13,14]. The aim of this study is to explore whether the modified model gives better prediction than that by the original FD model and to study the influence of hierarchical structure core on the performance of sandwich beams under impact loading.

The paper is organized as follows. In Section 2, the modified analytical model for the response of the sandwich beam under impact is presented. The developed analytical model is then compared against FEM simulations and the original FD predictions in the Section 3. In Section 4, the analytical model is then applied to study the effects of core relative density and core thickness on the impact resistance of sandwich beams. The influence of hierarchically designed cores on the impact performance of sandwich beams is also studied in Section 4. Finally, a few conclusions are drawn.

2. Analytical model for clamped sandwich beams subjected to impact loading

In this paper, the response of clamped sandwich beams subjected to impact loading is investigated (see, Fig. 1). The core can be either ordinary or hierarchical cellular structures, which is featured by compressive strength σ_{cY} in direction 1 and tensile strength σ_{lY} in direction 2. The cellular core has initial thickness c, and density ρ_c, with its stress-nominal strain relationship modeled by the R-P-P-L model [7] and the densification strain denoted as ε_D. The front face and rear face, which are made from the same material with plastic yielding stress σ_{ys}, Young's modulus E_s and density ρ_s, have equal thickness h and equal area mass denoted as m_f.Therefore, the area mass of the sandwich beam m can be expressed as $m = 2m_f + c\rho_c$.

To simplify the analysis, the impact loading is mimicked by a uniform initial velocity v_0 of the front face. Following FD [9], two stages of the deformation responses can be identified. The first stage is the core compaction stage (i.e., stage II in FD [9]). During this stage, the front face and compacted part of the core decelerate with velocity denoted as v_D at the time instant t, while the non-compacted part of the core and the rear face accelerate with velocity denoted as v_U at the time t. This process lasts until the front face, rear face and core achieve equal velocity v_f or until the core is totally compacted in which case additional collision of the front face and compacted core with the rear face occurs ending in the same common velocity v_f. In this process, the compaction wave front propagates with Lagrangian velocity v_s. Therefore, the length of the compacted part of the core increases while the length of the non-compacted part of the core decreases. The second stage is the retardation stage (i.e., stage III in FD [9]), during which the sandwich beam deforms as a whole with initial velocity v_f and the nominal compressing strain of the core as ε_c. Assumptions that no heat is generated in the core and the area of the core stay

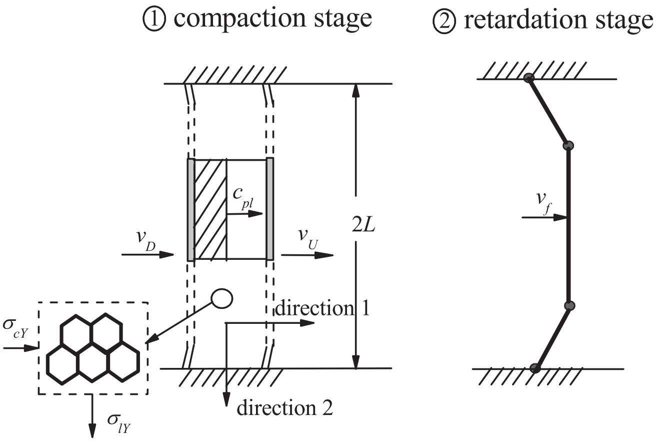

Fig. 1. Schematic of the two stages in the response of a sandwich beam under impulsive blast loads.