International Journal of Impact Engineering 47 (2012) 71–78

Contents lists available at SciVerse ScienceDirect

International Journal of Impact Engineering

journal homepage: www.elsevier.com/locate/ijimpeng

A constitutive description of elastomer behaviour at high strain rates – A strain-dependent relaxation time approach

H. Pouriayevali*, Y.B. Guo, V.P.W. Shim

Impact Mechanics Laboratory, Department of Mechanical Engineering, National University of Singapore, 9 Engineering Drive 1, Singapore 117576, Singapore

ARTICLE INFO

Article history:
Received 20 December 2011
Received in revised form
1 April 2012
Accepted 3 April 2012
Available online 7 April 2012

Keywords:
Relaxation time
High strain rate
Rubber
Visco-hyperelasticity

ABSTRACT

A strain-dependent relaxation time perspective is proposed for a visco-hyperelastic constitutive equation to describe the large compressive and tensile deformation response of incompressible elastomeric materials at high strain rates. The description comprises two components: the first characterizes quasi-static nonlinear response corresponding to hyperelasticity, using a polynomial strain energy density function, while the second is an integral form of first and captures rate sensitivity by incorporating a deformation-dependent relaxation time function. The model is applied to describe the response of six types of elastomer with different hardnesses, namely U50, U70 polyurethane rubber, SHA40, 60, 80 rubber, and Ethylene-Propylene-Diene-Monomer (EPDM) rubber. Material samples are subjected to quasi-static and dynamic loading using a universal testing machine and a Split Hopkinson Bar device respectively. The proposed equation is able to track the experimental responses and demonstrate the potential to predict the dynamic behaviour of elastomeric material over a range of strain rates.

1. Introduction

Elastomers are able to accommodate large deformations and possess damping characteristics, making them suitable for employment in the dissipation of kinetic energy associated with impacts and shocks. Therefore, analysis and modelling of the dynamic response of elastomers over a wide range of strains and strain rates are essential and will facilitate the use of computer simulation for designing products that incorporate elastomeric padding and components.

The stress–strain responses of elastomers generally exhibit nonlinear rate-dependent elastic behaviour associated with negligible residual strain after unloading from a large deformation [1,2]. The observed rate-dependence corresponds to the readjustment of molecular chains, whereby the applied load is accommodated through various relaxation processes (e.g. rearrangement, reorientation, uncoiling, etc., of chains). An elastomer contains a wide range of molecular chain lengths - e.g. in localized regions with relatively short chains, as well as long convolutions that span larger areas. These chains are cross-linked and spiral and entangle among themselves or with neighbours. They are also able to slip and rearrange to accommodate relaxation [3–5].

Tosaka et al. [6] suggested that when relatively small strains are applied to elastomers, the deformation is accommodated primarily within localized regions; deformation extends throughout the material as the strain is increased. Relaxation processes occur relatively rapidly within localized regions containing primarily shorter chains, while rearrangements of long convolutions require greater relaxation times [7]. Therefore, relaxation processes associated with smaller relaxation times, are activated at smaller strains, while relaxation for larger strains is linked to processes including long convolutions and greater relaxation times. Consequently, it is expected that the relaxation time is influenced by the degree of strain applied, because this determines which components of deformation corresponding to the various molecular chain length, are dominant.

When high rate deformation is applied, the material does not have sufficient time for all relaxation processes to be completed, and the material response is thus affected by incomplete rearrangement of chains. The relationship between the strain experienced and relaxation time is therefore investigated in this study.

Nonlinear-viscoelastic constitutive models have been proposed and developed to predict the behaviour of rubber-like materials [8–10]. These models can be classified according to two approaches: differential equations derived from micromechanics modelling [11–13], and history-integral models based on macromechanical theories associated with a fading-memory effect [14,15]. Arruda and Boyce [11] developed a rate-independent three-dimensional constitutive relationship based on an eight-chain representation of

* Corresponding author. Tel.: +65 6516 2228; fax: +65 6779 1459.
E-mail address: pouriayevali.habib@nus.edu.sg (H. Pouriayevali).

0734-743X/$ – see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijimpeng.2012.04.001

Nomenclature

A_{ij}	material parameters defining W ($i,j = 0, .., n$)
$\boldsymbol{B}$	left Cauchy-Green deformation tensor
B_{ij}	components of $\boldsymbol{B}$ ($i,j = 1, .., 3$)
$\boldsymbol{E}$	Green-Lagrange strain tensor
I_i	invariants of $\boldsymbol{B}$ ($i = 1, 2, 3$)
E_{ij}	components of $\boldsymbol{E}$ ($i,j = 1, .., 3$)
$\boldsymbol{F}$	deformation gradient tensor
X	position vector in the reference configuration
x	position vector in the deformed configuration
$-P_e\boldsymbol{I}$	hyperelastic undetermined pressure tensor
$-P_v\boldsymbol{I}$	viscoelastic undetermined pressure tensor
$\boldsymbol{\sigma}$	Cauchy stress tensor
$\boldsymbol{\Omega}$	frame-independent matrix function
t, τ	time
T	relaxation time
α_i	parameters used in hyperelastic description ($i = 1, 2$)
λ	stretch vector
λ_i	component of λ ($i = 1, 2, 3$)
W	strain energy density
β	relationship between elements A and B
ε	engineering strain tensor
ε_{11}	engineering strain in the direction of uniaxial loading
$\bar{\varepsilon}$	equivalent strain
$\varphi(\int d\bar{\varepsilon})$	deformation-dependent relaxation time function

Superscripts

e	hyperelastic
Ins	instantaneous
T	tensor transposition
v	viscoelastic

the molecular network structure of rubbers. Others extended the model to a hyper-viscoelastic equation to characterize the high strain rate behaviour of elastomers [1,16]. A set of integral-based equations was proposed by Yang et al., based on hereditary laws for high strain rate response [2]. This three-dimensional model comprises two parts: a hyperelastic and a viscoelastic component, and it has been employed by others to study elastomers [7,17–19]. A study by Shim et al. [17] examined both the compressive and tensile dynamic response of rubber, using Split Hopkinson Bar (SHB) devices. They modified the BKZ model [20] and increased the number of material parameters to characterize both the compressive and tensile rate-sensitive response of rubber.

Extensive experimental investigations into the properties of rubber, have been directed at eliciting material responses corresponding to relaxation, creep and deformation characteristics at low strain rates (<1/s) [11,21–23]. Experiments at high rates of loading (>100/s), performed using SHB devices [2,24–26], are quite challenging because of the relatively low mechanical impedance of rubber, whereby the stress propagated through such specimens into a metallic output bar is too small to be captured and modifications to SHB systems using metallic bars are necessary. In a study by Shim et al. [17], improvement in the output bar signal was attained by using polycarbonate bars instead of metallic ones, because of the smaller mechanical impedance difference between rubber and polycarbonate. They also investigated both the dynamic compressive and tensile response of rubber; these experimental results and the response of rubber reported in two previous works [27,28] are used in this study to validate the following proposed visco-hyperelastic constitutive model.

In essence, this investigation aims to propose a three-dimensional constitutive description that is able to define the high rate, large deformation compressive and tensile mechanical responses of approximately incompressible elastomeric material. The model is based on a macromechanics level approach, coupled with the novel proposition of a strain-dependent relaxation time. An accompanying objective is to formulate a model with a minimum number of material parameters and avoid the complexity of delving into the details of polymer structures. These prompt the development of a visco-hyperelastic description in integral form, based on the concept of fading-memory and comprising two components: the first corresponds to hyperelasticity based on a strain energy density function, expressed as a polynomial, to characterise the quasi-static nonlinear response; the second is an integral form of the first and incorporates a relaxation time function to capture rate sensitivity and deformation history dependence. Instead of employing a constant relaxation time, a novel approach of incorporating a deformation-dependent relaxation time function is adopted.

2. Constitutive model

The behaviour of an elastomer is considered to be amenable to idealization by two parallel elements, A and B, representing mechanical responses, as depicted schematically in Fig. 1. Element A defines the rate-independent quasi-static response and is modelled by a nonlinear spring that corresponds to incompressible hyperelasticity. Element B is associated with rate-dependent response, whereby molecular chains in the polymer encounter resistance to sudden stretching through entanglements with other long chains, constraints on chain mobility by crosslinks, and interactions between molecular chains. Depending on the restraints imposed on the chains, they rearrange to a more relaxed configuration after being deformed, and this takes a specific period of time characterized by a relaxation time T. Element B takes the idealized form of a nonlinear spring connected to a nonlinear dashpot.

The total stress is therefore, $\boldsymbol{\sigma}_{\text{total}} = \boldsymbol{\sigma}_A^{(\text{rate-independent})} + \boldsymbol{\sigma}_B^{(\text{rate-dependent})}$.

2.1. Incompressible visco-hyperelastic model

Consider a generic particle in a body, identified by its position vector X in a reference configuration, and by x in the deformed configuration. The deformation gradient and left Cauchy-Green deformation tensor are respectively $\boldsymbol{F} = \partial x/\partial X$ and $\boldsymbol{B} = \boldsymbol{F}\boldsymbol{F}^T$. The three invariants of $\boldsymbol{B}$ are: $I_1 = \text{tr}\,(\boldsymbol{B})$, $I_2 = [I_1^2 - \text{tr}\,(B^2)]/2$ and $I_3 = \det(\boldsymbol{B})$; assumption of incompressibility yields $I_3 = 1$. Following the analysis of Rivlin [29] and Yang et al. [2] for isotropic incompressible hyperelastic materials, the Cauchy stress for element

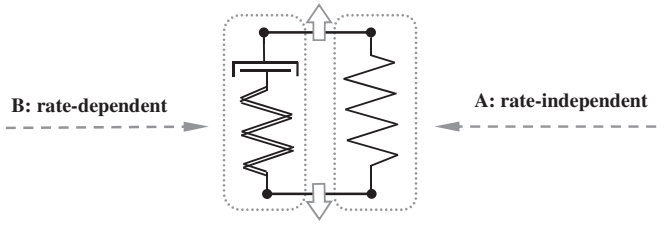

Fig. 1. Parallel mechanical elements A and B.