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Analytical formulas for penetration of a long rigid projectile including the effect of cavitation

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ABSTRACT

Analytical expressions for penetration of a long rigid projectile with a nose shape of an ovoid of Rankine into a semi-infinite incompressible elastic–perfectly-plastic target have been developed earlier. Using these expressions it is shown that the drag force applied by the target on the projectile can be approximated as a bilinear function of the square of the penetration velocity in terms of three non-dimensional constants $\{\Sigma_c, \alpha_c, \beta_{max}\}$. The value of Σ_c characterizes the constant value of the drag force for low penetration velocities. Cavitation (separation of the target material from the projectile's surface) first occurs when the penetration velocity reaches a value associated with α_c. The parameter β_{max} controls the dependence of the drag force on the square of the penetration velocity as the separation point on the projectile's surface approaches its tip. Analytical expressions for these constants are determined in terms of the material parameters of the target material. Also, a simple formula for the penetration depth is developed and a method is proposed for determining the constants $\{\Sigma_c, \alpha_c, \beta_{max}\}$ directly from experimental data.

1. Introduction

Backman and Goldsmith [3] document scientific interest in penetration mechanics from the beginning of the 19th century and a collection of experimental data can be found in [2]. Here, attention is limited to the case of a rigid projectile penetrating a semi-infinite incompressible elastic–perfectly-plastic target. Fig. 1 shows a typical axisymmetric projectile which has a nose that smoothly transitions to a circular cylinder of radius R. The projectile moves with velocity V in the negative $\mathbf{e}_3$ direction without rotation so the balance of linear momentum for the motion is given by

$$M\dot{V} = -F, \tag{1}$$

where M is the projectile's mass and F is the drag force applied in the positive $\mathbf{e}_3$ direction by the target material on the projectile. Letting s be the instantaneous depth of penetration and using the specifications

$$V = \dot{s}, \ V(0) = V_0, \ s(0) = 0, \tag{2}$$

it is convenient to introduce the normalized variables

$$\Sigma = \frac{F}{\pi R^2 Y}, \ \alpha = \frac{\rho V^2}{Y}, \ \alpha_0 = \frac{\rho V_0^2}{Y}, \ \lambda = \frac{2\rho\pi R^2 s}{M}. \tag{3}$$

Then, multiplying Eq. (1) by V yields

$$\dot{\alpha} = -\Sigma\dot{\lambda}, \quad \frac{d\alpha}{d\lambda} = -\Sigma. \tag{4}$$

In these expressions, V_0 is the impact velocity, and $\{\rho, Y\}$ are, respectively, the constant density and yield strength (in uniaxial stress) of the target. Moreover, Σ is the normalized drag force that the target applies to the projectile, α is a normalized inertia (kinetic energy) in the target and λ is the normalized instantaneous depth of penetration.

Hill [6] describes research done between May 1943 and March 1946 on cavitation during penetration of a rigid projectile into metal. He notes that due to melting at the projectile's surface the effects of friction are negligible. Consequently the traction vector $\mathbf{t}$ applied by the target material on the projectile can be approximated as a contact pressure P applied in the opposite direction to the outward unit normal $\mathbf{n}$ to the projectile's surface (see Fig. 1)

$$\mathbf{t} = -P\mathbf{n}. \tag{5}$$

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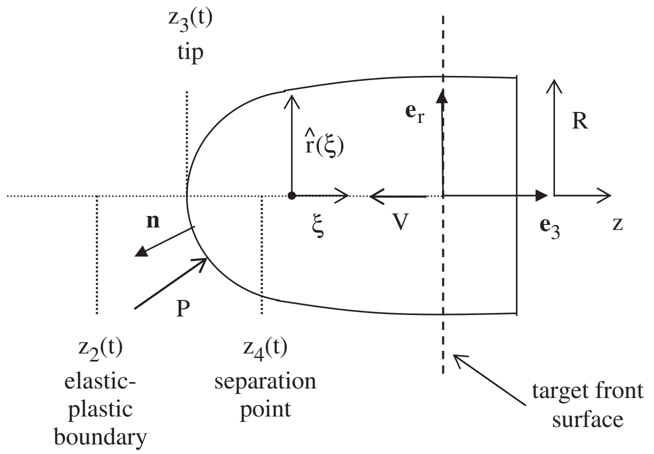

Fig. 1. Sketch of the nose region of a rigid projectile penetrating an incompressible elastic–perfectly plastic target.

Using this expression Σ is given by

$$\Sigma = \frac{2}{R^2 Y}\int_0^R P\, r\mathrm{d}r. \tag{6}$$

For long projectiles and deep penetration into thick targets the effect of the entrance phase, cavitation at the projectile's tail and the transient development of the plastic region can be neglected to obtain asymptotic expressions for P and Σ. Typical expressions for these quantities found in the literature can be written in the forms

$$P = Y(P_Y + P_I\alpha), \quad \Sigma = \Sigma_Y + \Sigma_I\alpha, \tag{7}$$

where $\{P_Y, \Sigma_Y\}$ characterize the effects of plasticity in the target and $\{P_I, \Sigma_I\}$ characterize the effects of inertia in the target.

In his analysis of the effects of cavitation Hill [6] suggested that P_Y in Eq. (7) is reasonably constant and that it can be estimated using static solutions for cavity expansion of the type developed in [5]. In particular, for expansion of a spherical cavity Bishop et al. [5] developed the expression

$$P_Y = \frac{2}{3}\left[1 + \ln\left(\frac{2G}{Y}\right)\right], \tag{8}$$

where use has been made of the relationship between the shear modulus G, Young's modulus E and Poisson's ratio ν given by

$$G = \frac{E}{2(1+\nu)}. \tag{9}$$

In addition, Hill [6] proposed an expression for P_I of the form

$$P_I = k\frac{\mathrm{d}}{\mathrm{d}\xi}\left(r\frac{\mathrm{d}r}{\mathrm{d}\xi}\right), \tag{10}$$

where k is a positive non-dimensional empirical constant and ξ is an axial coordinate measured from a material point in the projectile. For convenience, here ξ is specified by

$$\xi = z - z_3(t) - \frac{R}{2}, \quad -\frac{R}{2} \le \xi \le -\frac{R}{2} + L, \quad \dot{z}_3(t) = -V, \tag{11}$$

where $z_3(t)$ denotes the axial location of the projectile's tip (see Fig. 1) and L is its length. Since the projectile's nose smoothly transitions to a circular cylinder at its tail it follows that

$$\frac{\mathrm{d}r}{\mathrm{d}\xi} = 0 \quad \text{for } \xi = -\frac{R}{2} + L. \tag{12}$$

Moreover, Hill [6] confined attention to nose shapes for which

$$r\frac{\mathrm{d}r}{\mathrm{d}\xi} = 0 \quad \text{for } \xi = -\frac{R}{2}. \tag{13}$$

An important consequence of expression (10) is that for values of the penetration velocity V (related to α) less than a critical value V_c (related to α_c), the contact pressure remains non-negative over the entire curved surface of the projectile so the target material remains in contact with the projectile until its tail with no cavitation near the projectile's nose

$$P \ge 0 \quad \text{for } -\frac{R}{2} \le \xi \le -\frac{R}{2} + L \text{ with } \alpha < \alpha_c. \tag{14}$$

Next, assuming that P_Y is constant, Eq. (10) can be substituted into Eq. (6) and use can be made of Eqs. (6), (7), (12) and (13) to deduce that

$$\Sigma_Y = P_Y, \quad \Sigma_I = 0 \quad \text{for } \alpha < \alpha_c. \tag{15}$$

This means that for penetration velocities V less than the critical value V_c ($\alpha < \alpha_c$) the drag force is constant. Rosenberg and Dekel [9] confirmed this empirical result by analyzing details of a series of numerical simulations of nearly rigid projectiles with different nose shapes. Also, Rapoport and Rubin [8] used simplifications of expressions developed in [12] for a projectile with the shape of an ovoid of Rankine to prove this result analytically. Moreover, Rapoport and Rubin [8] pointed out that this constant value of drag is not expected using cavity expansion models for which the influence of inertia in the target on the drag force is always non-negative.

Next, using expressions (7) and (10) with P_Y constant, it follows that cavitation (separation) occurs at the location $\xi = \xi_s$ and the value $\alpha = \alpha_s$ when the pressure $P(x_s)$ vanishes so that

$$P(x_s) = Y[P_Y + P_I(x_s)\alpha_s] = 0 \Rightarrow \alpha_s = -\frac{P_Y}{P_I(x_s)} \text{ with}$$
$$\xi = \xi_s \text{ and } x = x_s, \tag{16}$$

where x is the normalized radius of the projectile

$$x = \frac{r}{R}. \tag{17}$$

The value α_c is the minimum value of α_s which occurs when $x = x_c$

$$\alpha_c = \min(\alpha_s) \quad \text{with } x = x_c. \tag{18}$$

In Eqs. (16) and (18) it has been convenient to introduce the non-dimensional variables $\{x_s, x_c\}$ to characterize the values of the radius at the point of cavitation. For higher values of the penetration velocity ($\alpha \ge \alpha_c$) the drag force is obtained by integrating only over the portion of the projectile's surface that is in contact with the target. Again, assuming that P_Y is constant it follows that both plasticity and inertia in the target influence the normalized drag force so that

$$\Sigma = \Sigma_Y(x_s) + \Sigma_I(x_s)\alpha \quad \text{for } \alpha \ge \alpha_c, \tag{19}$$

where the functions $\{\Sigma_Y, \Sigma_I\}$ depend on the normalized separation point x_s and are given by

$$\Sigma_Y(x_s) = x_s^2 P_Y, \quad \Sigma_I(x_s) = kx_s^2\left[\frac{\mathrm{d}r}{\mathrm{d}\xi}(\xi_s)\right]^2. \tag{20}$$

In particular, it is important to note that since x_s depends on the penetration velocity through the solution of Eq. (16), the drag force