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3D coupled multifield magneto-electro-elastic contact modelling

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A B S T R A C T

The present work deals with the general contact problem for coupled magneto-electro-elastic materials. Despite of the relevant technological applications, this topic of research has been treated only in some analytical works. But analytical solutions lack the generality of numerical methodologies, being restricted typically to simple geometries, loading conditions, idealized contact conditions and mostly taking into account transversely isotropic material symmetry with the symmetry axis normal to the contact surface. In this work, a numerical procedure for the three-dimensional frictional contact modelling of anisotropic coupled magneto-electro-elastic materials in presence of both electric and magnetic fields is presented for the first time. An orthotropic frictional law is considered, so anisotropy is present both in the bulk and in the surface. The methodology uses the boundary element method with explicit evaluation of the fundamental solutions in order to compute the magneto-electro-elastic influence coefficients. The contact model is based on an augmented Lagrangian formulation and it uses an iterative Uzawa scheme of resolution. Conducting, semi-conducting and insulated electric and/or magnetic indentation conditions, as well as orthotropic frictional contact conditions are considered. The methodology is validated by comparison with benchmark analytical solutions. Then, additional exploration examples are presented and discussed in detail, revealing that magneto-electric material coupling, conductivity contact conditions lead to a significant effect on the indentation force and contact pressure distributions. The influence of friction in electric and magnetic potential responses has been also proved to be very significant. Moreover, tangential loads exhibit an important influence both on the maximum values of the electric and magnetic potentials as well as on their distributions.

1. Introduction

The Magneto-Electro-Elastic (MEE) coupling present in multiferroic composites consisting of Piezoelectric (PE) and Piezomagnetic (PM) phases has been the focus of intensive research in last years, due to its wide and important technological applications at multiple scales, such as sensors, actuators, filters, oscillators, phase shifters, memory devices, and general smart structures [1]. On the other hand, the study of the contact problem is necessary in order to address problems like positioning of micro- and nanomechanisms and various functional devices, as well as in experimental testing and characterization of this kind of materials. Therefore, in this emerging topic of research, the pursuit of powerful and efficient capabilities for modelling this coupled multifield contact problem become crucial in order to predict and understand the underlying physics in the interaction of electromagnetic and mechanical processes.

Because of the mentioned interest, several analytical works devoted to model the contact problem in coupled MEE materials have recently appeared. One of the first works which study this problem was due to Hou et al. [2], who obtain the Boussinesq and Cerruti solutions and apply them to the frictional Hertz problem in transversely isotropic materials. Analytical solutions for the half-space indentation by rigid flat-ended, conical, and spherical punches in transversely isotropic MEE materials have been presented and discussed in detail by Chen et al. [3]. Indenters may be in any combination of conducting and insulating for both electric and magnetic fields. Both works are based on Fabrikant's method of potential theory for elastic materials [4]. Moving rigid punch solutions in two-dimensions (2D) have been considered by Zhou et al., both for frictionless [5,6] and frictional contact [7–11]. Ke and his coauthors [12–14] presented comprehensive studies on the contact mechanics of the homogeneous or functionally graded magneto-electro-elastic materials (FGMEEMs). They found that MEE fields on the normal surface, the in-plane surface and in the interior domain can be altered by adjusting the gradient index of FGMEEMs, which provides a very useful information to improve the resistance to contact damage and electro-magnetic failures at

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the contact surface of smart devices and smart structures.

Most of the analytical works modelling the MEE problem are extensions of previous efforts related to the modelling of the contact problem for materials with electro-mechanical coupling. For instance some of those mentioned above are based on the works by Ding et al. [15–17]. It should be mentioned also the work by Giannakopoulos and Suresh [18] who presented a theory of indentation of piezoelectric materials by using the Hankel transform for the three typical indenters (i.e. flat, conical, and spherical). See also further references in Rodríguez-Tembleque et al. [19]. For transversely isotropic PM materials, the frictionless axisymmetric indentation by flat rigid punch has been studied by Giannakopoulos and Parmaklis [20]; and the 2D exact solution of the singular integral equation corresponding to the indentation by a sliding rigid punch with flat or cylindrical profile has been presented by Zhou and Lee [21].

But all these works lack the generality of numerical methodologies, being restricted to simple geometries (a few axisymmetric indenters on a half-space), loading conditions, idealized contact conditions and taking into account almost uniquely transversely isotropic material symmetry with the material symmetry axis being normal to the contact surface. These limitations can be overtaken with advanced numerical formulations. Mathematical models in variational form for coupled electro-elastic frictional contact problems have been proposed (e.g. [22,23]); and some numerical schemes based in the Finite Element Method (FEM) have been implemented. Quasistatic 2D contact problems between solid-foundation and a PE material and an electro-visco-elastic material have been studied, for instance, by Sofonea et al. [24–27] under frictionless conditions; and by Sofonea et al. [28] incorporating isotropic frictional contact conditions. Michopoulos et al. [29–31] have studied the multiscale contact problem of magneto-electro-elastoplastic materials in the context of FEM, and such models have been validated against actual experiments. These works incorporate information about the roughness of the surfaces at the microscale (see [32–37]) and thermal fields are taken into account.

Recently, Rodríguez-Tembleque et al. [19] presented a Boundary Element Method (BEM) formulation to study 3D frictional contact of piezoelectric bodies in the presence of electric fields. However, to the best of the authors' knowledge, no BEM formulation for solving the coupled MEE contact problem is available in the literature.

BEM is an alternative particularly advantageous over other numerical techniques when it is used for solving the contact problem in finite, semi-finite or infinite domains. The ability of the BEM to accurately represent steep solution gradients is well known and the reduction in the degrees of freedom becomes particularly attractive to handle the concerning problem, since for multifield materials the number of degrees of freedom per node notably increases. Note however that, contrary to the FEM, conventional BEM formulations result in fully populated non-symmetric system of equations.

The aim of this work is to present and to implement a BEM formulation for the 3D coupled modelling of the sliding contact interaction between anisotropic magneto-electro-elastic materials in presence of both electric and magnetic fields. An orthotropic frictional law is considered, so anisotropy is present both in the bulk and in the surface. The paper is organized as follows: in Section 2, the basic governing equations are presented. Non-linear mechanical and magneto-electrical contact conditions are presented in Sections 3 and 4. The literature on BEM formulations is quite extensive, so in Section 5.1 we briefly present the basic ideas of the BEM with emphasis in the explicit evaluation procedure for the fundamental solutions. MEE contact discrete equations are presented as an algebraic equation system in Section 5.2. Then, the solution method is presented in Section 6. The methodology is validated by comparison with benchmark analytical solutions in Section 7, where additional exploration examples are presented and discussed in detail. We close the paper with some concluding remarks.

2. Coupled magneto-electro-elastic equations

Let consider a 3D region $\Omega \subset \mathbb{R}^3$ with a piecewise smooth boundary $\partial\Omega$ occupied by a homogeneous MEE anisotropic material, in reference at a Cartesian coordinate system (x_i) $(i = 1, 2, 3)$. Small deformations are considered, so the infinitesimal strain tensor $\boldsymbol{\gamma}$, the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{H}$ are obtained, respectively, from derivatives of the displacements $\boldsymbol{u}$, the electric potential $\boldsymbol{\varphi}$ and the magnetic potential $\boldsymbol{\psi}$ as

$$\gamma_{ij} = (u_{i,j} + u_{j,i})/2 \quad \text{in } \Omega,$$
$$E_i = -\varphi_{,i} \quad \text{in } \Omega,$$
$$H_i = -\psi_{,i} \quad \text{in } \Omega. \tag{1}$$

Assuming static loading conditions and the absence of volume forces and free electric charges, the mechanical stress $\boldsymbol{\sigma}$, the electric displacement $\boldsymbol{D}$ and the magnetic induction $\boldsymbol{B}$ are divergence-free fields, that is,

$$\sigma_{ij,j} = 0 \quad \text{in } \Omega, \tag{2}$$

$$D_{i,i} = 0 \quad \text{in } \Omega, \tag{3}$$

$$B_{i,i} = 0 \quad \text{in } \Omega, \tag{4}$$

where repeated dummy indices indicate summation. In linear MEE materials, the elastic, electric and magnetic fields are coupled through the constitutive law

$$\sigma_{ij} = c_{ijkl}\gamma_{kl} - e_{lij}E_l - q_{lij}H_l \quad \text{in } \Omega, \tag{5}$$

$$D_i = e_{ikl}\gamma_{kl} + \epsilon_{il}E_l + \lambda_{il}H_l \quad \text{in } \Omega, \tag{6}$$

$$B_i = q_{ikl}\gamma_{kl} + \lambda_{il}E_l + \mu_{il}H_l \quad \text{in } \Omega, \tag{7}$$

where $\boldsymbol{c}$, $\boldsymbol{\epsilon}$ and $\boldsymbol{\mu}$ denote the components of the elastic stiffness tensor, the dielectric permittivity tensor and the magnetic permeabilities tensor, respectively; $\boldsymbol{e}$, $\boldsymbol{q}$ and λ are the PE, PM and ME coupling coefficients, respectively. These tensors satisfy the following symmetries:

$$c_{ijkl} = c_{jikl} = c_{ijlk} = c_{klij}, \quad e_{kij} = e_{kji}, \quad q_{kij} = q_{kji}, \tag{8}$$

$$\epsilon_{kl} = \epsilon_{lk}, \quad \lambda_{kl} = \lambda_{lk}, \quad \mu_{kl} = \mu_{lk}.$$

Moreover, the elastic constant, dielectric permittivity, magnetic permeability tensors are positive definite; and the PE, PM and ME coupling tensor are positive semi-definite.

Given a MEE region Ω, three partitions of the boundary $\partial\Omega$ are considered to define the mechanical, the electrical and the magnetic boundary conditions (see Fig. 1). The first one divides $\partial\Omega$ into three disjoint parts such that $\partial\Omega = \partial\Omega^u \cup \partial\Omega^t \cup \partial\Omega^c$. Here, $\partial\boldsymbol{\Omega}^u$ denotes the boundary on which displacements $\tilde{u}_i$ are prescribed; $\partial\boldsymbol{\Omega}^t$ denotes the boundary on which the tractions $\tilde{t}_i = \sigma_{ij}\nu_j$ are