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Modeling of tension–compression asymmetry and orthotropy on metallic materials: Numerical implementation and validation

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ABSTRACT

The details concerning the implementation of the yield criterion developed by Cazacu et al. 2006 (CPB06) [1], which accounts for both tension–compression asymmetry and orthotropy of the plastic flow, in the fully implicit FE solver DD3IMP (contraction of 'Deep Drawing 3-D IMPlicit') are presented in this work. The implemented constitutive model is extensively described, including the analytical first and second order derivatives required to the stress update algorithm. A set of anisotropy parameters describing the mechanical behavior of two metallic materials at room temperature, namely Zirconium and AZ31-Mg alloy, are identified with the DD3MAT (contraction for 'Deep Drawing 3-D MATerial') in-house code (Alves, 2004) [2]. The anisotropy parameters are identified for both the CPB06 and the Cazacu and Barlat (2001) (CB2001) [3] yield criteria, in order to emphasize the importance and role of the strength differential effect. The results clearly show that the CPB06 yield criterion is able to accurately describe both the in-plane anisotropy and tension–compression asymmetry, as well a different anisotropic behavior in uniaxial tension and uniaxial compression. The numerical simulation of a four-point bending test is performed, considering different orientations of the beam, i.e. of the hard/soft to deform direction relatively to the load direction, allowing to validate the implementation. The results obtained with the CPB06 show its ability to describe with accuracy the strain fields in the beam's central cross-section, the distribution of the tensile and compressive layers and, consequently, the shift of the neutral layer. The comparison with the results obtained with CB2001 indicates that the strength differential effect affects the final deformed shape of the beam, particularly for materials exhibiting strong tension–compression asymmetry.

1. Introduction

Currently, plastic forming of metals with hexagonal closed packed (HCP) structure poses tremendous challenges due to their low ductility at room temperature and their unusual deformation characteristics, i.e. very pronounced anisotropy with strong asymmetry between tensile and compressive behavior. Unlike cubic metals (both face centered cubic (FCC) and body centered cubic (BCC)), hexagonal metals deform due to the activation of mechanical twinning or non-Schmid type slip at single crystal level. In contrast to slip, twinning is a directional shear mechanism: in general, shear in one direction can produce twinning while shear in the opposite direction cannot [1,3–6]. This causes the strength differential (SD) effect, or tension–compression asymmetry. However, although less pronounced, the SD effect is also present in materials with cubic structure [7].

Until recent years, the SD effect has been neglected with the major developments being made in macroscopic plasticity models for materials with cubic structure. In fact, in most numerical analysis of metal forming processes, the yield surface is assumed to possess a point-symmetry with respect to the center, such that a stress state and its reverse state have the same absolute value [3,8–11].

Cazacu et al. (2004) extended an isotropic yield function to the anisotropic case through invariants generalizing, able to describe both the materials anisotropy and tension–compression asymmetry [4]. Later, Cazacu et al. presented another yield function (CPB06) that enables describing the asymmetric yielding between tension and compression due to twining as well as in-plane anisotropy through a linear transformation, with a fourth-order tensor, of the deviatoric stress tensor [1]. Some authors have later adopted several linear transformations in order to more accurately capture in-plane anisotropy [6,12]. Tuninetti et al. characterized the mechanical behavior of a Ti-6Al-4V alloy using uniaxial tension, uniaxial compression, simple shear and plane strain tests in three orthogonal material directions, identifying the CPB06 yield criterion parameters by inverse modeling of the axial strain field of

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Nomenclature

$\mathbf{A}$	set of parameters used in the objective function according to the selected yield criterion
$\mathbf{C}$	anisotropy parameters tensor for the CPB06 yield criterion
$\mathbf{C}^{e}$	Elastic modulus
$\mathbf{C}^{ep}$	Elastoplastic modulus
$c, a_1, \ldots, a_6, b_1, \ldots, b_{11}$	CB2001 yield criterion anisotropy parameters
$C_{11}, C_{22}, C_{33}, C_{44}, C_{55}, C_{66}, C_{23}, C_{13}, C_{12}, k, a$	CPB06 yield criterion parameters
$\mathbf{D}$	Strain rate tensor
$\mathbf{D}^{e}$	elastic strain rate tensor
$\mathbf{D}^{p}$	plastic strain rate tensor
E	Young's modulus
$\mathbf{E}$	Green-Lagrange strain tensor
$\mathbf{F}$	Deformation gradient tensor
G	Shear modulus
H'	isotropic hardening modulus
$\mathbf{I}_4$	forth-order identity tensor
J_2^0, J_3^0	second and third generalized invariants of $\boldsymbol{\Sigma}$
$\mathbf{Q}$	second derivative of the equivalent stress $\bar{\sigma}$ in order to the effective stress tensor, $\boldsymbol{\Sigma}$
$\mathbf{R}$	orthogonal plastic rotation tensor
r_θ	anisotropy coefficient for direction θ with the rolling direction
s_1, s_2, s_3	principal values of $\mathbf{s} = C\boldsymbol{\sigma}'$
$\mathbf{V}$	first derivative of the equivalent stress $\bar{\sigma}$ in order to the effective stress tensor, $\boldsymbol{\Sigma}$
$\mathbf{W}$	total spin tensor
$w_{\sigma_\theta^T}, w_{\sigma_\theta^C}, w_{r_\theta}, w_{\sigma_b}, w_{r_b}$	weighting factors used in the objective function
Y	flow stress
α	material state parameter
β	material state parameter
γ	generalized middle point rule parameter
$\Delta\boldsymbol{\varepsilon}$	total strain increment tensor
$\Delta\boldsymbol{\varepsilon}^{p}$	plastic strain increment tensor
$\bar{\varepsilon}^{p}$	equivalent plastic strain
θ	angle with the rolling direction
$\dot{\lambda}$	plastic multiplier
μ	Lamé parameter
υ	Poisson ratio
$\boldsymbol{\Sigma}$	effective stress tensor
$\boldsymbol{\sigma}$	Cauchy stress tensor
$\boldsymbol{\sigma}'$	Deviatoric Cauchy stress tensor
$\dot{\boldsymbol{\sigma}}^{J}$	Jaumann derivative of the Cauchy stress tensor
$\bar{\sigma}$	equivalent stress
σ_θ^T	tensile yield stress for direction θ with the rolling direction
σ_θ^C	compression yield stress for direction θ with the rolling direction
σ_b^T	equibiaxial yield stress in tension
σ_b^C	equibiaxial yield stress in compression
Φ	Yield surface and plastic potential

compression specimens in the three orthogonal directions of the material [13]. In fact, several works have been made for the characterization of both titanium [14,15] and magnesium alloys [16,17], both presenting a HCP crystal structure.

The modeling of the SD effects allowed to point a new interpretation of Swift effects. Considering the isotropic form of the CPB06 yield criterion, Cazacu et al. showed that a slight difference between the uniaxial yield stresses in tension and compression leads to irreversible length changes under monotonic free-end testing conditions [18]. Revil-Baudard et al. show that there is a correlation between the Swift phenomenon in torsion and the stress–strain behavior in uniaxial tension and compression [17]. Chandola et al. showed that to explain and accurately predict the room-temperature torsional response of a strongly textured AZ31 Mg material it is necessary to account for the combined effects of anisotropy and tension–compression asymmetry at polycrystalline level [16].

Revil-Baudard and Cazacu numerically assessed the influence of the tension–compression asymmetry of the plastic flow in the matrix on void evolution and the location of the zone corresponding to maximum damage, in round tensile specimens subject to uniaxial tension [19]. Alves and Cazacu [20] used a detailed micromechanical finite-element analyses of three-dimensional unit cells, considering a spherical void at its center, with the plastic flow of the matrix described by the isotropic form of the CPB06 yield criterion. It was shown that there is a strong correlation between SD effects in the plastic flow of the matrix, arising from its dependence on the third stress invariant, with void evolution, and ultimately the ductility of porous metallic polycrystals. The combined effect of both the tension–compression asymmetry and anisotropic behavior of the material was also studied [17]. Also, due to non-negligible twinning activity accompanied by grain reorientation and highly directional grain interactions, the influence of the texture evolution on the hardening behavior of HCP materials cannot be neglected, even for the simplest monotonic loading paths [21]. In fact, Plunkett et al. shows a way of describing distortional hardening based on the evolution, with plastic strain, of the CPB06 yield *locus* [5].

The aim of this work is to evaluate the importance of taking into account the tension–compression asymmetry in the constitutive model. In this context, an associated flow rule is considered, neglecting the evolution of the shape of the yield surface with plastic work. The manuscript starts by presenting the details concerning the constitutive model, including the non-quadratic yield criterion proposed by Cazacu, Plunkett and Barlat and its implementation into the implicit in-house FE code DD3IMP, which has been continuously developed and optimized to simulate sheet metal forming processes [22,23]. In this context, the main aspects of the implementation of the CPB06 yield criterion in an implicit finite element code are presented, including the general aspects of the linear transformation operating in the deviatoric stress space. The validation of the implemented model is performed with the numerical simulation of a beam subjected to a four-point-bending test, along two different directions. The results obtained with the CPB06 yield criterion are compared with the ones of the Cazacu and Barlat (CB2001) [3] yield criterion, since the later enables an accurate description of the in-plane anisotropy while neglecting the SD effect. This allows analyzing the strain fields evolution in the beam's cross section, as well as the final shape of the beam when considering, or not, the SD effect.

2. Constitutive model

The constitutive equation that models the materials' mechanical behavior establishes the relationship between the most relevant state variables characterizing the continuum medium. In the following, it is assumed that constitutive modeling is