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Mode selection for reduced order modeling of mechanical systems excited at resonance

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ABSTRACT

Welding, food cutting, atomizing, cleaning and deagglomerating are just a few common uses for ultrasonic resonators, which are carefully designed to operate excited in resonance. Finite element analysis is nearly always adopted for predicting and improving resonator performances. Large number of small elements are usually needed to guarantee accuracy. As a consequence, models have typically very large dimensions, and hence considerable computational and ill conditioning problems arise. Model reduction techniques can be extremely useful to keep model dimensions to a minimum. In this paper a new ranking method, called Interior Mode Ranking (IMR), is introduced for the selection of the interior normal modes in the Craig Bampton reduction technique, which is one of the most popular model reduction methods, often available in commercial finite element software packages. The IMR method allows ranking the interior modes analytically by comparing the contributions provided by the interior modes of the subsystem with constrained boundary conditions to the dynamics of interest of the complete system (with actual boundary conditions). The method is general and can be applied to any resonator in the reduction at the system level. Here it is employed to obtain an accurate reduced-order model of an ultrasonic welding bar horn. The results achieved by the method are compared with those yielded by other ranking techniques. The comparison shows that the IMR method outperforms the other ranking techniques and leads to accurate representations of the excited modes.

1. Introduction

The use of large-dimensional models is usually needed to get accurate dynamic representations of complex vibrating systems. These models, typically synthesized through finite elements (FE) or experimental modal analysis, are often very bulky and require considerable computational efforts, which can prevent their use in simulation [1] and real-time control [2]. Additionally, large dimensional models are often numerically ill conditioned and therefore they cannot be effectively exploited for design optimization (e.g. for the solution of inverse structural modification problems [3,4]). Therefore, reduced order models can be very useful in model-based design (see e.g. [5,6]), simulation (see e.g. [2]), control (see e.g. [7,8]) and estimation (see e.g. [9]). Indeed, not only can such models be computationally more reliable and efficient, but their use also simplifies the experimental identification of model parameters by just requiring accurate investigations in a restricted frequency range [10].

Several approaches to model reduction have been presented in literature. Generally speaking either the reduction of different models of different substructures to be coupled has been addressed (the so called "reduction at the component level") or the reduction of the complete model of the complete system ("reduction at the system level") [11]. The latter is the approach that should be preferred whenever reduced order models are employed for numerical simulations, for the synthesis of model-based controllers or for model-based design and optimization. Indeed, in these fields it is usually desirable getting the best trade-off between accuracy and size, at the cost of an increase in modeling complexity. Therefore, first of all, an accurate full-order model is often synthesized, and then the less relevant vibration modes are truncated in accordance with model specifications (i.e. type of model coordinates and frequencies of interest). Indeed, assuming independence between subsystems (i.e. model reduction by means of "Component Mode Synthesis" [12]) often affects negatively model correctness, by resulting in poor design [13] or controllers with small robustness margins. Therefore, while this assumption makes sense for the purpose of getting faster coupling

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tests between subsystems, it is reasonable reducing models at the full system level whenever accuracy should be boosted.

The model reduction methods proposed to date can be basically divided into three main categories, according to the variables employed in the model: non physical, physical and semi physical subspace reduction methods [14]. Clearly, the selection of the most suitable method for performing model reduction depends on several parameters to be evaluated, and is also often a subjective matter since each technique has strengths and weakness.

Non physical reduction methods, such as the Modal Truncation, the Krylov Subspace Method [15] and the Balanced Truncation [16], originate from control theory. These techniques are often unsuitable to model reduction in mechanical systems, since the physical degrees of freedom (dofs) of the full order models are replaced by non physical coordinates and therefore they do not retain their straightforward physical interpretation. Moreover, these techniques may be ill conditioned for large scale models [7]. Physical coordinates are of interest, for instance, for coupling a system with other systems designed either in the mechanical domain or in other domains, such as the coupling between electro-mechanical systems in multi-physics simulations or when simulating complex plants (e.g. manufacturing plants). Physical coordinates are also of interest when modifications to physical system parameters should be computed through structural modification techniques. Conversely, physical subspace reduction methods are widespread in structural mechanics, since they lead to intuitive model representations in physical coordinates. Among the most relevant, dynamic condensation [17], improved reduction system [18], Guyan condensation [19] are to be mentioned. Generally speaking, physical reduction methods provide good approximations for static analyzes or for dynamic analyzes at low frequencies, but may be not accurate when high frequency motion is contemplated [20]. Finally, semi physical methods are often the most suitable to mechanical systems, since the reduced models obtained with these techniques can approximate both the static and the dynamic behavior of mechanical systems accurately and also retain some physical coordinates of interest. In semi physical methods the system behavior is represented through a set of physical coordinates, the so called master dofs, and through a reduced set of non physical coordinates (whose definition depends on the specific boundary conditions set on the physical dofs). On the basis of the boundary conditions set on the master dofs for computing the non physical coordinates, the semi physical techniques are grouped into fixed interface methods [21,22], free interface methods [23–26], hybrid interface methods [24] and loaded interface methods [27]. Among these methods the fixed interface Craig-Bampton (CB) method [21] is by far the most employed. Indeed, due to a simple and straightforward formulation of the reduction process, combined with good overall performances, the CB approach is widely used in structural dynamics, and is by far the most popular reduction method in the multibody field. Additionally, it is available in most commercial FE codes, which often implement only such a method among those proposed in literature. Generally speaking, the CB method is basically considered a standard framework for model reduction with hybrid coordinates.

An open issue for all the above mentioned semi physical methods is how the reduced set of interior vibrational modes (to be retained in the reduced order models) should be chosen in order to ensure a correct representation of the system dynamics, while keeping model dimensions to a minimum. Following a widespread approach, here referred to as sorting based on eigenfrequency (SBE), only the modes with the lowest natural frequencies are usually retained [28]. As a rule of thumb, the modes up to 1.5–2 times the highest frequency of interest are retained, due to a supposed higher energy content of the lowest frequency modes. However, SBE is not based on any optimality criterion and may not always guarantee the achievement of valid results [29], especially when the reduced model should describe the behavior of a system in a prescribed frequency range. A few methods have been developed in literature to rank and select the interior vibrational modes to be retained. For example, in the case of CB reduction, the Effective Interface Mass (EIM) method [30,31], the Optimal Modal Reduction (OMR) method [29,32] and the "Component Mode Synthesis χ" (CMSχ) method [33]. Basically, all these ranking methods evaluate how the interior normal modes interact with the system interface, on the basis of some coupling terms. They are therefore suitable for model reduction at the component level. Unfortunately, all these methods neglect any specification on the frequencies at which reduced models should be accurate, which is an essential requirement in some mechanical systems, such as resonant systems (or resonators). Resonators are designed to operate excited at resonance and hence to vibrate at a specific frequency with a prescribed spatial shape. Popular examples are ultrasonic sonotrodes, which are designed to vibrate in a purely longitudinal mode at a specific excitation frequency [34]. The dynamic behavior of such systems mostly relies on one or just a few vibrational modes which should be represented accurately. The same issue arises in those systems whose dynamics is dominated by a reduced subset of vibrational modes, such as those with higher observability or controllability.

The main aim of this paper is to propose a novel approach, called Interior Mode Ranking (IMR), which has been developed to rank and select the interior normal modes ensuring an accurate description of the full order model dynamics of a resonant system at a specific set of frequencies of interest, by representing its dominant modes. The method is perfectly suited to the designed optimization of ultrasonic resonators. Indeed, for these systems the availability of reduced models is an essential need for performing numerical simulations (also coupling mechanical and electro-mechanical models when simulating actuators [35]), or for developing model-based design, especially in the case of resonators with complex geometries (e.g. heavily contoured resonators or composite resonators) that require very large three-dimensional models. Additionally, a resonator must be tuned at a specific frequency. If such a frequency is not matched correctly, resonator optimization becomes necessary as well as the availability of a reduced model if inverse structural modification is to be employed. Although the proposed method is targeted resonators, it has a general validity. Indeed, it can also be applied to perform reduction at system level of systems with dominant modes. For example this could be the case of controlled systems, which just have some dominant vibration modes, i.e. the ones observable and controllable. Therefore, on the one hand the proposed method has a practical interest, since commercial FE codes implementing the CB method usually allow designers to select the interior modes to be retained. On the other, it has a theoretical interest since interior mode selection is not trivial. Indeed, there is no straightforward relation between the modes of the complete system and the interior modes, i.e the modes of the one with fixed boundary conditions on the master dofs.

In the proposed IMR method the interior modes are ranked and selected on the basis of some newly defined participation coefficients, which represent such a relation. An analytical formulation is provided for the aforementioned participation coefficients. The coefficients allow comparatively evaluating the different contributions provided by the interior modes to the excited modes of the complete system. In particular, the coefficients take into account the contributions of the interior modes to the full system modes on the basis of modal shapes, excitations and natural frequencies.

The theory developed is here validated with reference to a single slot sonotrode, identical to the one used in [34] and whose shape recalls the one of the horns employed for ultrasonic