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Transient dynamic response of generally-shaped arches based on a GDQ-time-stepping method

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ABSTRACT

This paper deals with the in-plane dynamic modeling of generally shaped arches with a varying cross-section in undamaged or damaged configuration, under different boundary conditions and external forces. The Generalized Differential Quadrature (GDQ) method is herein applied to solve numerically the problem without passing through any variational formulation, but solving directly the governing equations of motion in strong form. The main purpose of the work is to obtain a computationally efficient higher-order method for solving time integration problems. The total time interval is discretized in time steps and the GDQ method is applied to solve the initial value problem within each time step. At each time interval, a linear algebraic equation system has to be solved. A simple and efficient implementation scheme is presented. A wide GDQ-based numerical investigation is performed to study the linear dynamics of the arch with different geometries, boundary conditions and external forces. The numerical results based on the application of the GDQ method are compared with those ones provided by the Newmark method. A very good agreement is found between the two numerical approaches, which demonstrates the performance and feasibility of the proposed GDQ-time-stepping algorithm for transient dynamics.

1. Introduction

The analysis of the static and dynamic behaviour of curved beams, arches and vaults, has been very attractive for a long time, because of the use of these structural elements for many applications. After some early studies on the topic, as provided by Hartog [1] and Love [2], a large amount of works in the literature has investigated the structural behaviour of arched structures both theoretically [3–20] and practically [21,22]. Despite the large applications of structural dynamics in many engineering fields, an analytical approach to the problem may be sometimes very complicated or unfeasible. The arched structures indeed, usually change their modes of vibration during the excitation, with each mode being governed by a different set of differential relations with their associated boundary conditions. On the other hand, some experimental investigations are required for validation purposes in the field, although they are usually difficult and costly to perform. These limitations have enhanced the usefulness of numerical modeling to investigate the dynamic response of arched structures, mainly based on Finite Element Method (FEM), Rayleigh–Ritz approach, Galerkin method, and cell discretizations (see [23–28], among others). Possible numerical drawbacks of FEM and complex algorithms, however, can be overcome by using the Generalized Differential Quadrature (GDQ) method, as discussed accurately in the book by Shu [29] and in a recent review paper [30]. This approach is well known in the literature for its accuracy, stability and reliability despite the limited number of grid points adopted for numerical applications [31–56]. More specifically, the GDQ method turns out to be really useful for studying the free vibrations of undamaged and damaged arches with different shapes as widely demonstrated in the literature [17,57–60]. A first application of the GDQ method, for example, is given by Liu and Wu [17] to study the free vibrations of inextensible circular arches with varying cross-sections and different boundary conditions. Karami and Malekzadeh [4] later removed the hypothesis of inextensibility of the central axis, including the rotary inertia in their differential quadrature formulation. Based on these two pioneering works, Viola and Tornabene [58] applied the GDQ approach to study the free harmonic vibrations of non-uniform arches in undamaged and damaged configurations, while modelling the cracked cross-section with a mass-less and elastic rotational hinge. This allows for a rotational discontinuity proportionally to the applied bending moment at the cracked cross-section. Based on their formulation, for each portion of the arch

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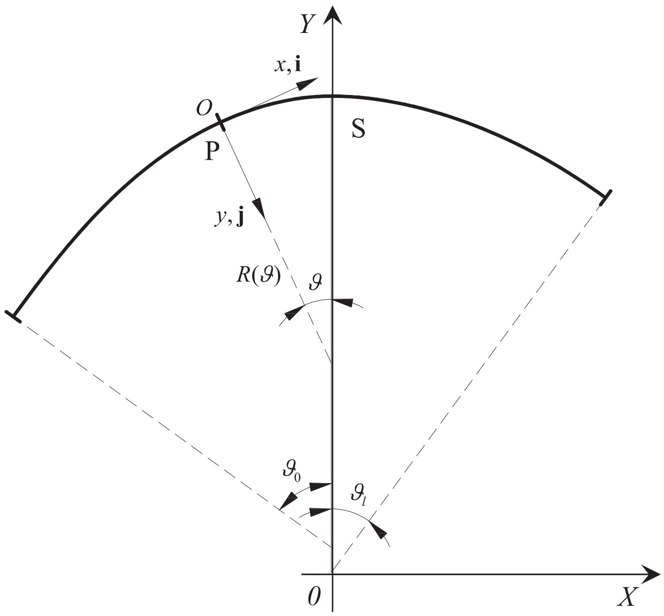

Fig. 1. Generally-shaped arch with the local and global reference configurations.

between the crack and the support, a set of equations of motion was written without any simplifying hypothesis. Thus, the natural frequencies and modal shapes of the structural element was determined by enforcing the boundary conditions and jump conditions across the damaged section. The work was then extended to multi-stepped arches with circular [59] and parabolic [60] shapes, accounting for the development of an arbitrary number of cracks inside the structure. In these cases, the damage was allowed to be located inside an arch portion, or at the interface between two different portions. Recently, the GDQ method is combined with the mapping technique, typical of FEM, to solve complex problems with mechanical and geometrical discontinuities by using a strong formulation. The accuracy and stability features of this approach are proven in the works [61–72] both in the 2D and 1D structural problems. The Strong Formulation Finite Element Method (SFEM) was introduced for the first time in the literature [58,59] to solve the governing equations of some arched structures with discontinuities. Based on results, the applied method revealed to be a powerful tool, in presence of damaged configurations or concentrated loadings within arched structures. Beyond the numerical applications of the GDQ method for vibration problems of continuous systems and arched elements, and increasing attention in the literature has been devoted to solve transient problems by means of higher-order methods [73–79]. The computational efficiency of numerical solvers, indeed, is known to strongly depend on the chosen time integration scheme. Thus, an efficient and reliable solver is one of the prerequisites for a successful solution when solving dynamic problems on time-varying domains. In this context, a GDQ-based approach was successfully applied by Tomasiello [73] to study the non-linear dynamics of continuous systems (i.e. a simply supported beam on a non-linear Winkler soil, and a slender clamped-hinged beam), including the internal resonances and other phenomena generally neglected in the usual reduction to a single-degree-of-freedom oscillator. Civalek [74] studied the dynamic behaviour of thin isotropic plates by coupling discrete singular convolution and harmonic differential quadrature methods to discretize the spatial and temporal domains, respectively. Maleki et al. [75] analysed the linear transient behaviour of laminated composite plates by means of the GDQ method under different loading and boundary conditions. Hong [76] applied the GDQ method to compute the linear transient response of thermal stresses and centre displacement in laminated magnetostrictive plates under thermal vibration. Peng et al. [77] adopted a semi-analytical perturbation differential quadrature method for a geometrically non-linear vibration analysis of circular plates. In addition, Civalek [78] performed a geometrically non-linear dynamic analysis of thin doubly curved isotropic shells supported by an elastic foundation (i.e. a Winkler foundation), based on a combination of harmonic differential quadrature-finite difference methods. The non-linear transient response of moderately thick laminated composite shallow shells was successfully studied by Kurtaran [79] by applying the

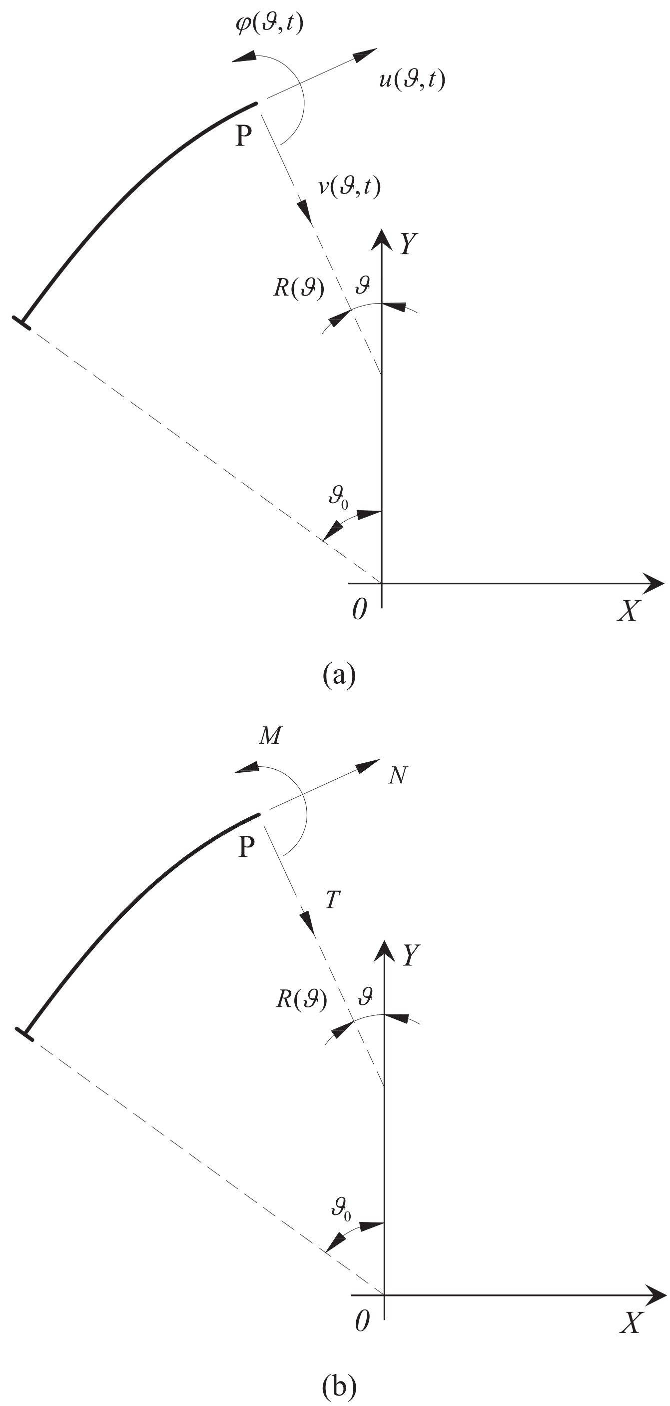

Fig. 2. Generally-shaped arch with the displacement (a) and internal force (b) components.