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Sensitivity analysis of crack detection in beams by wavelet technique

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Abstract

This paper examines the sensitivity of wavelet technique in the detection of cracks in beam structures. Specifically, the effects of different crack characteristics, boundary conditions, and wavelet functions employed are investigated. Crack characteristics studied include the length, orientation and width of slit. The two different boundary conditions considered are simply supported and fixed end support, and the two types of wavelets compared in this study are the Haar and Gabor wavelets. The results show that the wavelet transform is a useful tool in detection of cracks in beam structures. The dimension of the crack projected along the longitudinal direction can be deduced from the analysis. The method is sensitive to the curvature of the deflection profile and is a function of the support condition. For detection of discrete cracks, Haar wavelets exhibit superior performance. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Crack detection; Beams; Haar and Gabor wavelets; Sensitivity study; Finite elements

1. Introduction

Amongst the many damage detection methods, the use of modal analysis of vibration signals in time domain is most common and extensively researched. This includes works by Schultz and Warwick [1], Adams et al. [2], Banks et al. [3] and Narkis [4]. Modal based methods have certain shortcomings. Even though a baseline is not always available [5], the vibration responses of a structure before and after damage has occurred are desired. In addition, a complete dynamic analysis of the whole structure is often performed to locate and quantify the damage. Banks et al. [3] also argued that it may be necessary to include the exact geometry of the damage for meaningful results.

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Damage detection using wavelet transform is a recent area of research in structural health monitoring. Wavelet-based methods do not require the analysis of the complete structure and neither do they require any knowledge of the material properties nor the prior stress states of the structure. Although studies have shown that wavelet techniques are highly feasible for damage detection [5–13], the treatments presented are rather fundamental and some specific issues have not been addressed. This study attempts to examine the suitability of the wavelet technique that makes use of deflection profiles of beams. Of interest is the sensitivity of this technique to different damage characteristics, boundary conditions and types of wavelets. The scope of this study is limited to static loads only from which spatial data are obtained for the analysis.

2. Wavelets and wavelet transform

In employing wavelet techniques, two important mathematical entities must be introduced, namely wavelet functions and wavelet transform. A complex-valued function $\psi(x)$ that is localised in both time and frequency domains is used to create a family of wavelets $\psi_{a,b}(x)$, where a and b are real numbers that dilate (scale) and translate the function $\psi(x)$, respectively [13]. These $\psi(x)$ functions are known as mother wavelets and can be continuous or discrete. For the continuous case, the complex-valued wavelets generated from the mother wavelet are given by

$$\psi_{a,b}(x) = |a|^{-1/2}\psi\left(\frac{x-b}{a}\right), \tag{1}$$

where a is the real-valued dilation parameter and b the real-valued translation parameter. For the discrete case, the wavelets take the form

$$\psi_{j,k}(x) = 2^{j/2}\psi(2^j x - k), \tag{2}$$

where ψ is the mother wavelet, j the integer dilation (scale) index and k the integer translation (position) index.

For a given signal (temporal or spatial), the wavelet transform can be obtained by integrating the product of the signal function and the complex conjugate of the wavelet function. The resulting set of wavelet coefficients is a measure of the correlation between the wavelet and the corresponding segment of the signal. Mathematically, the continuous wavelet transform, $Wf_{a,b}$, of a signal $f(x)$ is defined as

$$Wf_{a,b} = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} f(x)\overline{\psi\left(\frac{x-b}{a}\right)}\,\mathrm{d}x = \int_{-\infty}^{\infty} f(x)\overline{\psi_{a,b}(x)}\,\mathrm{d}x, \tag{3}$$

whereas the discrete wavelet transform, $Wf_{j,k}$, is defined as

$$Wf_{j,k} = 2^{j/2}\int_{-\infty}^{\infty} f(x)\overline{\psi(2^j x - k)}\,\mathrm{d}x = \int_{-\infty}^{\infty} f(x)\overline{\psi_{j,k}(x)}\,\mathrm{d}x. \tag{4}$$

The overbar in the above equations denotes the complex conjugate of the function under it. In this study, both types of wavelet transform are used to analyse the simulated data. There are many wavelet functions that have been developed, such as the Daubechies family of wavelets