International Journal of Mechanical Sciences 111-112 (2016) 55–64

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Rotating plasticity and nonshakedown collapse modes for elastic–plastic bodies under cyclic loads

Canh V. Le [a], T.D. Tran [b], D.C. Pham [c,*]

[a] Department of Civil Engineering, International University – VNU HCM, Vietnam
[b] Faculty of Construction & Electricity, Ho Chi Minh City Open University, 97 Vo Van Tan Street, Ho Chi Minh City, Vietnam
[c] Institute of Mechanics – VAST, 264 Doi Can, Hanoi, Vietnam

ARTICLE INFO

Article history:
Received 12 December 2015
Received in revised form
27 March 2016
Accepted 5 April 2016
Available online 6 April 2016

Keywords:
Elastic–plastic material
Shakedown
Cyclic loading
Plastic collapse modes
Rotating plasticity
Alternating plasticity
Incremental plasticity

ABSTRACT

Different plastic collapse modes may have different effects on the overall behaviour and load-bearing capacity of an elastic–plastic structure subjected to variable loads, and they may even be determined by different material plastic constants (for general plastic hardening materials). Both lower bound static and upper bound reduced kinematic approaches have been implemented with appropriate finite element realizations and mathematical programming techniques to study the nonshakedown modes for elastic plastic bodies under cyclic loads. For sufficiently complex structure and loading program, it has been firstly demonstrated that an elastic–plastic body may fail by rotating plasticity collapse rather than the simpler alternating plasticity one among other possible modes. That and other results also lead to interesting problems for further studies.

1. Introduction

An elastic–plastic structure subjected to variable and cyclic loads, after possible initial limited plastic deformations, may eventually shake down to some residual stress state, from which it would respond elastically to the loads afterwards. Otherwise, the structure should collapse instantaneously, incrementally, or fail by alternating plasticity (the phrase “alternating plasticity” shall be substituted by the more general one “bounded cyclic plasticity” – as a result of this study). Shakedown limits may be determined through an incremental analysis following a particular loading history, or path-independently via the shakedown kinematic (upper bound) or static (lower bound) theorems [1–8,10–13,15,17–24].

Because of the complexity of the shakedown theorems, which are the nonlinear optimization problems, methods have been developed to study the separated nonshakedown collapse modes such as the instantaneous (plastic limit), incremental plasticity, and alternating plasticity ones [5,7–10,16,24-26,28,32]. Separation of collapse modes does not only simplify the shakedown analysis. It is necessary because different collapse modes have different effects on the load-bearing capacity of the structures [9], and they may be determined by different material plastic constants [12–15]. Semi-analytical methods have been developed to derive from the upper and lower bound shakedown theorems simple estimates of the nonshakedown loads. However for generally complex structures under complicated loading programs one needs efficient numerical methods to solve the respective nonlinear programming problems.

For engineering applications, the shakedown theorems lead to large-scale nonlinear convex optimization problems with large numbers of variables and constraints. Direct iterative optimization algorithms have been developed to provide solution of such the non-linear programming [35,42,47]. However, the primal-dual interior-point method has been found to be more efficient and robust [33,38–41,45]. The algorithm has been extended to both static and kinematic shakedown analysis problems [18,23,31,44,46]. Taking advantages of the algorithm, Tran et al. [24] have developed an effective computational method for kinematic separated shakedown analysis, in which the trial fields obtained from the plastic limit analysis problems corresponding to possible external load combinations were used to solve the incremental plasticity mode. In this paper, the algorithm will be further developed for static separated shakedown analysis. The non-linear yield criterion is formulated as a quadratic conic constraint, making sure that the obtained optimization problem can be solved using highly efficient solvers.

* Corresponding author.
E-mail addresses: lvcanh@hcmiu.edu.vn (C.V. Le), pdchinh@imech.ac.vn (D.C. Pham).

http://dx.doi.org/10.1016/j.ijmecsci.2016.04.001
0020-7403/

For most problems considered in the literature, the non-shakedown limit curve is the lower envelope of the incremental collapse and bounded cyclic (or alternating) plasticity collapse curves. The computation of the bounded cyclic plasticity limit should involve elastic stresses only. However, general formulation applied to compute the alternating plasticity limit for various yield criteria is not available, but only for plane problems governed by von Mises criterion [16]. In this paper, an analytical formulation for determination of the alternating plasticity limit will be statically derived for different non-linear yield criteria which can be cast in the form of L^2 norm, such as the 2D and 3D Drucker–Prager, Mohr–Coulomb and Nielsen criteria [37], and hence enabling broader applications of the separated shakedown analysis. Moreover, the rotating plasticity, which in the mathematical sense is a generalization of the alternating plasticity collapse [12,13,15], will also be derived analytically for general time-independent stress center, and yield criteria. In this paper, it has been firstly demonstrated that an elastic–plastic body may fail by rotating plasticity collapse rather than the simpler alternating plasticity one among other possible modes. The rotating plasticity mode is the imaginary mode, in which plastic deformations do not increase unrestrictively in magnitudes. The plastic strain tensor change the directions in the strain space during the process (rotate).

The layout of the paper is as follows. In the next section some theoretical results from both kinematic and static shakedown theorems are resumed, with some particular questions raised. In Section 3 the discrete formulations of the shakedown problems and respective solution methods are presented, which are followed by particular numerical implementations in Section 4. Discussions on the obtained results and possible further studies are made in the last section.

2. Shakedown theorems and collapse modes

2.1. Kinematic upper bound approach

Let $\boldsymbol{\sigma}^e(\mathbf{x}, t)$ denote the fictitious elastic stress response of the body V to external agencies over a period of time $(\mathbf{x} \in V,\ t \in [0, T])$ under the assumption of perfectly elastic behaviour, called a loading process (history). The actions of all kinds of external agencies upon V can be expressed explicitly through $\boldsymbol{\sigma}^e$. At every point $\mathbf{x} \in V$, the elastic stress response $\boldsymbol{\sigma}^e(\mathbf{x}, t)$ is confined to a bounded time-independent domain with prescribed limits in the stress space, called a local loading domain $\mathcal{L}_x$. As a field over V, $\boldsymbol{\sigma}^e(\mathbf{x}, t)$ belongs to the time-independent global loading domain $\mathcal{L}$:

$$\mathcal{L} = \left\{ \boldsymbol{\sigma}^e \middle| \boldsymbol{\sigma}^e(\mathbf{x}, t) \in \mathcal{L}_x,\ \ \mathbf{x} \in V,\ \ t \in \left[0, T\right] \right\} \tag{1}$$

Shakedown of a body in $\mathcal{L}$ means it shakes down for all possible loading histories $\boldsymbol{\sigma}^e(\mathbf{x}, t) \in \mathcal{L}$.

Let $\mathcal{A}$ be the set of admissible compatible-end-cycle (deviatoric) plastic strain rate fields $\mathbf{e}^p$ over time cycles $0 \le t \le T$:

$$\mathcal{A} = \{\mathbf{e}^p | \boldsymbol{\varepsilon}^p = \int_0^T \mathbf{e}^p dt \in C\} \tag{2}$$

where C is the set of strain fields that are both deviatoric and compatible on V;

$$\mathcal{A}_0 = \left\{ \mathbf{e}^p | \boldsymbol{\varepsilon}^p = \int_0^T \mathbf{e}^p\, dt = \mathbf{0} \right\} \tag{3}$$

is a subset of $\mathcal{A}$, which contains the plastic strain rate fields that vanish at the end of the cycle.

Let k_s be the shakedown safety factor: at $k_s > 1$ the structure will shake down, while it will not at $k_s < 1$, and $k_s = 1$ defines the boundary of the shakedown domain. We consider, generally, the elastic–plastic nonlinear kinematic hardening materials that satisfy the positive hysteresis postulate and are bounded by the initial yield stress σ_Y^I and ultimate yield stress σ_Y^U [12,13,15]. Then, the shakedown kinematic theorem can be stated as the optimization problem:

$$k_s^{-1} = \max\{U, C\}, \tag{4}$$

where

$$U = \sup_{\mathbf{e}^p \in \mathcal{A};\, \boldsymbol{\sigma}^e \in \mathcal{L}} \frac{\int_0^T dt \int_V \boldsymbol{\sigma}^e : \mathbf{e}^p\, dV}{\int_0^T dt \int_V D_u\left(\mathbf{e}^p\right) dV}, \tag{5}$$

$$C = \sup_{\mathbf{x} \in V;\, \mathbf{e}^p \in \mathcal{A}_0;\, \boldsymbol{\sigma}^e \in \mathcal{L}} \frac{\int_0^T \boldsymbol{\sigma}^e : \mathbf{e}^p\, dt}{\int_0^T D_i\left(\mathbf{e}^p\right) dt}; \tag{6}$$

$D(\mathbf{e}^p) = \boldsymbol{\sigma} : \mathbf{e}^p$ is the dissipation function determined by the yield stress σ_Y and the respective yield criterion; e.g. for a Mises material we have

$$D(\mathbf{e}^p) = \sqrt{2/3}\, \sigma_Y (\mathbf{e}^p : \mathbf{e}^p)^{1/2} \tag{7}$$

$D_u(\mathbf{e}^p)$ and $D_i(\mathbf{e}^p)$ are the particular expressions of $D(\mathbf{e}^p)$ with the ultimate yield stress and initial yield stress taking the place of σ_Y, respectively.

For more-convenient numerical implementations, the formulations (4)–(6) can be represented alternatively as

$$k_s = \min\{\bar{U}, \bar{C}\}, \tag{8}$$

where

$$\bar{U} = \inf_{\mathbf{e}^p \in \mathcal{A};\, \boldsymbol{\sigma}^e \in \mathcal{L}} \frac{\int_0^T dt \int_V D_u\left(\mathbf{e}^p\right) dV}{\int_0^T dt \int_V \boldsymbol{\sigma}^e : \mathbf{e}^p\, dV} \tag{9}$$

[here the implicit condition that the denominator $\int_0^T dt \int_V \boldsymbol{\sigma}^e : \mathbf{e}^p\, dV$ be positive is assumed, otherwise the expression $\inf(\cdot)$ should be trivial $-\infty$, which is physically meaningless],

$$\bar{C} = \inf_{\mathbf{x} \in V;\, \mathbf{e}^p \in \mathcal{A}_0;\, \boldsymbol{\sigma}^e \in \mathcal{L}} \frac{\int_0^T D_i\left(\mathbf{e}^p\right) dt}{\int_0^T \boldsymbol{\sigma}^e : \mathbf{e}^p\, dt} \tag{10}$$

(with the implicit condition that the denominator $\int_0^T dt \boldsymbol{\sigma}^e : \mathbf{e}^p\, dV$ be positive). At $\bar{C} = 1 < \bar{U}$ (or $C = 1 > U$) the elastic plastic body should fail by bounded cyclic plasticity, while at $\bar{U} = 1 < \bar{C}$ (or $U = 1 > C$) it is expected to collapse incrementally. Eq. (9) [or (5)] is just the expression of Koiter's shakedown kinematic theorem for elastic perfectly plastic bodies with the yield stress $\sigma_Y = \sigma_Y^U$; and it involves generally global compatible-end-cycle plastic strain rate fields $\mathbf{e}^p \in \mathcal{A}$. In the meantime the bounded cyclic plasticity mode (10) [or (6)] with the yield stress $\sigma_Y = \sigma_Y^I$ is strictly local and appears simpler. When $\sigma_Y^U = \sigma_Y^I$ the specific case of elastic-perfectly plastic material is obtained.

From the kinematic theorem (8)–(10) the following much simpler upper bound reduced kinematic formulation can be deduced [8,11–13,15,24]

$$k_s \le k_{sA} = \min\{\bar{I}, \bar{A}\}, \tag{11}$$

where