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# A theoretical investigation on low-velocity impact response of a curved sandwich beam

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ABSTRACT

In this paper, two different theoretical approaches are applied to study the low-velocity impact behavior of a curved sandwich beam. At first, a new high order model is developed for curved sandwich beams. Then, the accuracy of this high order model in evaluating the dynamic response and stress results is verified by the layerwise theory. The Ritz method is used to discretize the governing equations and the Runge–Kutta numerical integration method is applied to solve the set of discretized equations. Studies show that the effect of curvature angle is in conjunction with boundary conditions. For a simply supported beam with circumferentially free ends, the contact force decreases and contact time and maximum deflection increase when the curvature angle increases. While this scenario is completely reverse for a beam with circumferentially fixed ends in which the beam becomes stiffer by increasing the curvature angle.



## 1. Introduction

Sandwich structures consist of two stiff and high strength facesheets separated by a thick and lightweight core. These structures frequently encounter low-velocity impact during services such as maintenance damage or drop tool, collision between service cars, cargo and structures. Therefore, it seems important to develop a model that could accurately predict the behavior of sandwiches under low-velocity impact. Since sandwich structures have various failure modes, this model should be able to assess the shear and normal stresses in the facesheets, core and their interfaces [1]. Chai and Zhu [2] conducted a review on low velocity impact of sandwich structures. Their review paper shows that several simple impact models such as spring-mass and energy balance methods have been proposed for flat sandwich structures and the effects of various parameters such as impact mass, geometry of impactor and boundary conditions have been studied. Sandwich structures may be used as curved panels or beams in various applications. Although many researchers have studied the low-velocity impact response of flat beams, curved sandwich beams need more understanding and attention.

A few studies have experimentally investigated the low-velocity impact behavior of curved sandwich panels. Moody et al. [3] studied the residual compression strength of impacted curved sandwich panels. They reported that increasing the curvature angle increases the beam's transverse stiffness and damage size. Baba et al. [4] conducted experimental impact tests on curved sandwich beams with debonding and fixed boundary conditions. Their tests show that increasing the curvature angle decreases the maximum deflection and increases the contact force.

High order theories are convenient methods to analyze the sandwich structures. Frostig et al. [5] proposed the high order theory for sandwich panel (HSAPT) that considers the transverse flexibility of the core. In this HSAPT, it is assumed that the longitudinal normal stress of the core is negligible. By applying this assumption under static equilibrium condition, quadratic and cubic polynomial distributions are obtained for the core's transverse and longitudinal displacements. The Euler–Bernoulli theory is used for the facesheets and finally the compatibility conditions are applied at the facesheet/core interfaces. Frostig and Baruch [6] investigated the free vibration response of a flat sandwich beam and assumed that the core's velocity and acceleration components have the same distributions found for the core's displacements. The numerical results show that this model could also be employed in dynamic problems. Yang and Qiao [7] studied the impact behavior of a sandwich beam using the HSAPT. They employed the Hertzian contact law to assess the impact load and used the Ritz method to solve the model equations. Their results were verified by simulation in LS-DYNA finite element software. Yang and Qiao [8] applied the finite difference method to the HSAPT in order to analyze the effect of various boundary conditions in a sandwich beam with asymmetric lay-up. Also they have

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calculated the stress propagation through the sandwich beam and used a failure criterion to evaluate the mode, location and time of the failure. They found that employing the HSAPT provides a proper assessment of the generated stresses and induced damage during impact. Malekzadeh et al. [9] developed the high order model by introducing the first order shear deformation theory to facesheets and employed this model to study the response of sandwich panels under low-velocity impacts of multiple masses. They used the Hertzian contact model to evaluate the contact force and validated their model with reported experimental results. Khalili et al. [10] modeled a sandwich beam as a discrete three degrees of freedom dynamic system with equivalent masses and spring. The dynamic response is based on the high order theory and the effects of some important physical and geometrical parameters have been investigated. Phan [11] proposed the extended high order theory (EHSAPT) which regards the axial rigidity of the core. She used the EHSAPT to study the low-velocity impact response of a flat sandwich beam. Salami et al. [12] assumed a bilinear elasto-plastic behavior for the core and applied the improved high order sandwich panel theory (IHSAPT) to study the nonlinear bending response of sandwich beams.

Frostig [13] presented a high order theory to analyze the bending of a curved sandwich beam. He applied the same assumption of negligible circumferential stress and found a pattern for the core's displacement components. Bozhevolnaya and Frostig [14] investigated the free vibration response of a curved beam and used the pattern obtained in the bending problem for the displacement components. But they used linear distributions for the core's velocity and acceleration components. Bozhevolnaya and Sun [15] applied linear patterns for the core displacement components similar to its velocity and acceleration components. Their analyses show that the boundary condition has a significant influence on the free vibration response of a curved beam.

The limitations of equivalent single layer theories for thick laminates have motivated the researchers to develop refined plate theories. To this end, a generalized plate theory was proposed by Reddy et al. [16] and improved by Barbero and Reddy [17] as a layerwise theory. In Reddy's layerwise theory, all physical layers of the beam, plate or shell are being discretized into a number of mathematical layers in the thickness direction by assuming a unique displacement field within each mathematical layer [18]. This solution approach with the linear Lagrangian interpolation function through the thickness displacements has been employed to many problems of composite laminates [19–22]. Since this theory takes into account through the thickness displacement of the structure, it can be used to analyze sandwich panels and hybrid structures. Based on layerwise mechanics, Oh [23] investigated the dynamic characteristic of a cylindrical hybrid panel containing a viscoelastic layer. Using the layerwise theory with linear interpolation functions, Afshin et al. [24]

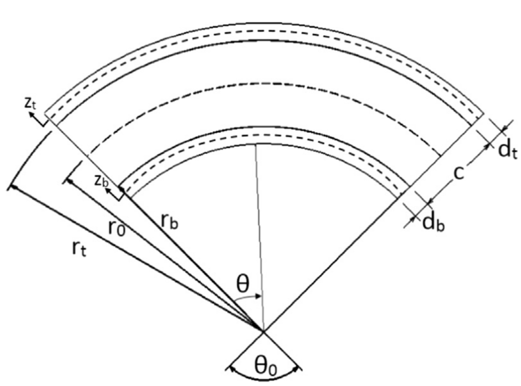


**Fig. 1.** Schematic representation of a curved sandwich beam.

studied the free edge effects in a cylindrical sandwich panel with a flexible core and laminated composite facesheets. They also employed this theory to validate the results of the high order sandwich theory for static and damped vibration analysis in a cylindrical sandwich panel [25,26]. A high-order layerwise theory was used by Plagianakos and Saravanos [27] to analyze the free vibration of thick composites and sandwich plates. Using this theory, Theofanis et al. [28] investigated the low energy impact of sandwich composite plates with piezoelectric layers.

The literature survey indicates that research works are quite limited on the low-velocity impact response of the curved sandwich panels. In this study, a new high order model is developed for curved sandwich beams and its results are compared with those of the layerwise theory. The comparison includes both impact response and the core's stress components results. Also, the effects of curvature angle and boundary conditions on impact response are presented.

## 2. Analytical modeling

### 2.1. Mathematical formulation based on high order model

Fig. 1 shows a curved sandwich beam of width $b$ and arc-length $L$. The beam consists of a core with height $c$ and two facesheets with thickness $d_t$ and $d_b$. The subscript $t$, $b$ and $c$ stand for the top facesheet, bottom facesheet and the core, respectively.

The displacement fields of the facesheets are supposed to follow the Euler–Bernoulli curved beam theory:

$$w_j(z_j,\theta)=w_{0j}(\theta)$$
$$u_j(z_j,\theta)=u_{0j}(\theta)+z_j\beta_j(\theta)$$
$$\beta_j=\frac{u_{0j}(\theta)-w_j(\theta)_{,\theta}}{r_j}\ (j=t,b) \tag{1}$$

where $w_{0j}$ and $u_{0j}$ are the radial and circumferential displacement components; $r$ and $\theta$ are the radial and circumferential coordinates; and $z_j$ is the radial distance from the centerline of each facesheet.

Assuming small deformation, the circumferential strain of the facesheets may be defined as:

$$\epsilon_{\theta\theta j}=\ \epsilon_{0j}(\theta)+z_j\kappa_j(\theta)$$
$$\epsilon_{0j}(\theta)=\frac{u_{0j}(\theta)_{,\theta}+w_j(\theta)}{r_j}$$
$$\kappa_j(\theta)=\frac{u_{0j}(\theta)_{,\theta}-w_j(\theta)_{,\theta\theta}}{{r_j}^2}(j=t,b) \tag{2}$$

where $\kappa_j$ and $\epsilon_{0j}$ are the curvature and circumferential strain of the facesheets' centerlines, respectively.

The facesheets are considered isotropic, therefore the circumferential stress of the facesheets can be stated in term of circumferential strain:

$$\sigma_{\theta\theta j}=E_f\epsilon_{\theta\theta j} \tag{3}$$

where $E_f$ is Young's modulus of the facesheets.

The authors propose that the transverse and circumferential displacements of the core are quadratic polynomials of the radial distance:

$$w_c(r,\theta)=w_{0c}+w_{1c}(r-r_0)+w_{2c}(r-r_0)^2$$
$$u_c(r,\theta)=u_{0c}+u_{1c}(r-r_0)+u_{2c}(r-r_0)^2 \tag{4}$$

where $r_0$ is the radial coordinate of the core's centerline. Six variables were defined in the above equation to describe the core displacement components. These six variables are reduced to two variables by applying displacement compatibility conditions at the interfaces. $w_{0c}$ and $u_{0c}$ are assumed to be independent variables; hence the remaining four unknown variables in Eq. (4) can be written in terms