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Pattern formation of elastic waves and energy localization due to elastic gratings

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ABSTRACT

Elastic wave propagation through diffraction gratings is studied numerically in the plane strain setting. The interaction of the waves with periodically ordered elastic inclusions leads to a self-imaging Talbot effect for the wavelength equal or close to the grating size. The energy localization is observed at the vicinity of inclusions in the case of elastic gratings. Such a localization is absent in the case of rigid gratings.

1. Introduction

Wave motion in solids is an extremely important physical phenomenon due to wide range of applications. The propagation of mechanical waves can be controlled via scattering induced by a material's structure. Given the high frequencies and high values of excitations used in contemporary technology, the material properties must be very clearly determined up to smaller scales and an internal structure of materials. The need to tailor materials able to meet various conditions is obvious. That is why during the last two decades, the attention to man-made metamaterials has been tremendously increased [41,20,27,44]. This is understandable because metamaterials are characterized by their properties beyond those of conventional engineering materials and therefore their practical applications display new qualities for technology. The reader is referred to more detailed overviews on this topic [34,35,25].

It is not surprising that the wave propagation in metamaterials cannot rely on classical continuum mechanics based on the homogeneity of materials. Indeed, the wave propagation in solids with inhomogeneities (inclusions) or microstructured solids at various scales has also been studied intensively based on various assumptions about the internal structure of the material [30,12,15] and various mathematical models were derived (see e.g. Engelbrecht and Berezovski [14]). From the physical viewpoint, the most important feature of waves in microstructured materials is the interaction of waves with inhomogeneities which is the source of wave dispersion, diffraction and interference. However, as in optics, one of the basic problems to be solved is the diffraction of waves on inhomogeneities. In light and atom optics, the diffraction of waves is a well-known and well-studied phenomenon, both for near-field (Fresnel diffraction) and far-field (Fraunhofer diffraction) zones. In solid mechanics the interest to the diffraction started due to the practical importance of the dynamic stress concentration on obstacles [31] and nondestructive testing [22]. The earlier theoretical studies related to the elastic wave diffraction on inclusions were shadowed by analytical difficulties [19,1]. It is remarkable, however, that the elastic counterpart of the well-known Talbot effect in optics [39] is not so largely studied. Discovered by Henry Fox Talbot in 1836, the phenomenon involves the diffraction of a plane wave through a grating. As a result of such a process, a regular diffraction structure, called the Talbot carpet, appears which reproduces the structure of the grating at multiples of a certain distance. This distance is now called the Talbot length. Lord Rayleigh [36] proved that the appearance of the Talbot structure was a consequence of the Fresnel diffraction. He also determined the Talbot length $z_T = 2a^2/\lambda$, where a is the period of the grating and λ is the wavelength of the incident periodic wave.

The interest to the Talbot effect, i.e. to the diffraction through a grating is recently increased due to novel possible applications of the physical phenomena related to the diffraction: for example, atom lithography [27], quantum and optical carpets [11], electron spin effect [40], effects of metallic gratings [38], phononic crystals [24,17], behavior of metamaterials [43], etc. Clearly the Talbot effect in solids needs more detailed analysis because this is a basic case of the

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diffraction phenomenon. The demonstration of the elastic Talbot effect [10] was performed in full analogy with optical case [26], i.e., under the Kirchhoff assumption that distances from the aperture are larger than wavelengths and the grating is rigid. However, it is practically impossible to implement a perfectly rigid grating into an elastic material. It is worth, therefore, to investigate wave patterns appearing in a more suitable elastic grating case. It is clear that following the theory of elasticity both the longitudinal and transverse waves must be taken into account in the diffraction pattern.

In spite of the linearity of governing equations of classical elasticity, it was possible to construct solutions to diffraction problems only for rather simple problems like, for example, the diffraction from a single rigid barrier in an elastic medium by using the Wiener–Hopf method [19], the interaction of an impact wave with a rigid circular disc [23] and the interaction of an impact wave with a rigid plane inclusion [33].

The situation is changed with the growth of computational power and the progress in numerical techniques. The development of finite difference [3], finite element [21], and finite volume [28] algorithms for wave propagation in solids resulted in the numerous applications to nondestructive testing [42,18,2]. An example of an effective numerical analysis is a study of the guided waves at a periodic array of coplanar slits using Bloch harmonics [17]. Design of new materials like phononic crystals [34] demands even more accurate prediction of the wave propagation in structured solids.

The advantage of numerical simulation is its generality, capable of predicting accurate wave fields for any composite with arbitrarily distributed scatterers. In the case of elastic wave propagation, the wave field can be simulated accurately by solving the elastodynamic equations for the matrix and the scatterers respectively. In what follows we present the results of numerical simulations of elastic wave propagation for simple geometry of substructure with different material properties. We limit ourselves by the plane strain case since it is sufficient to demonstrate basic effects. We do not apply any kind of homogenization; all computations are performed directly for given materials.

In Section 2 the numerical procedure is described briefly. The reference wave pattern due to the rigid grating is reviewed in Section 3. Section 4 is devoted to the results of numerical simulations of wave patterns due to the elastic grating. Transmittance and the influence of wavelength are analyzed in Sections 5 and 6, respectively. An unexpected result for the energy localization is reported in Section 7. Finally, in Section 8, conclusions are given stressing the emergence of wave patterns and the localization of energy. The governing equations and its dimensionless form are presented in Appendix A. The important problem of boundary conditions is explained in Appendix B.

2. Numerical procedure

The governing system of equations (see Appendix A) is solved numerically by means of the conservative finite-volume wave-propagation algorithm, which was proposed by LeVeque [28,29] and modified for the application to front propagation by Berezovski et al. [6], Berezovski and Maugin [8,9]. The algorithm was successfully applied for the wave propagation simulation in inhomogeneous solids [7].

The modification of the wave-propagation algorithm [9] is based on the non-equilibrium jump relations at the boundaries between computational cells. The main idea in the construction of the numerical algorithm is the consideration of every computational cell as a thermodynamic system [32]. Since we cannot expect that this thermodynamic system is in equilibrium, its local equilibrium state is described by averaged values of field quantities. The use of cell averages is a standard procedure in the finite-volume methods. What is non-standard that is the introduction into consideration so-called “excess quantities” in the spirit of the thermodynamics of discrete systems [32].

Excess quantities represent the difference between values of true and averaged quantities [7]:

$$v_i = \overline{v}_i + V_i, \quad \sigma_{ij} = \overline{\sigma}_{ij} + \Sigma_{ij}. \tag{1}$$

Here v_i are components of the velocity vector, σ_{ij} are components of the stress tensor, overbars denote averaged quantities, and capital letters correspond to excess quantities.

Though excess quantities are determined formally everywhere inside computational cells, we need to know only their values at the boundaries of the cells, where they play the role of numerical fluxes that describe the interactions between neighboring cells [5]. These excess quantities are calculated by means of jump relations at the boundaries between cells [7]. It should be emphasized that jump relations used here provide the continuity of unknown fields at the boundaries between computational cells. The advantage of the algorithm is that every discontinuity in parameters is taken into account by the exact solution of the Riemann problem at each interface between discrete elements. The reflection and transmission of waves at each interface are handled automatically for any inhomogeneous media. The applied algorithm is conservative, stable up to the Courant number equal to 1, high-order accurate, and thermodynamically consistent [7,8].

3. Wave pattern due to rigid grating

As it was shown recently [10], the well-known Talbot effect in optics can be observed also in the case of elastic waves. The corresponding simulations, however, were performed for the case of perfectly rigid gratings. Here we demonstrate first a case where the size of rigid inclusions is equal to each other and to the distance between them. This scenario allows us to consider the problem independently from the length scales (see Appendix A). The geometry of the problem is shown in Fig. 1a. The grating is placed at 100 space steps from the left boundary.

The monochromatic plane wave is excited at the left boundary of the computational domain. The wavelength of the incident wave is equal to the size of the grating (20 space steps in the discretized computational domain). Boundary conditions at lateral boundaries are periodic (like Eqs. (47)–(48)). At the right boundary, the non-reflective boundary conditions (like Eqs. (45)–(46)) are applied. Note that in order to model rigid inclusions, all velocities and stresses are prescribed to be zero inside the inclusions. Additionally, boundary conditions for fixed boundary (similar to Eq. (B.4)) are prescribed at each side of the inclusion. Calculations performed up to 1400 time steps to avoid the influence of any reflection from the left boundary which is placed virtually at 500 space steps upstream the grating. It must be stressed that in this 2D elastic case both longitudinal and transverse wave exist.

The contour plots for the normal stress field along the longitudinal axis are shown in Fig. 1. The self-imaging Talbot carpet in the case of rigid grating is clearly seen in the contour plot (Fig. 1b). The emergent pattern is similar to this presented in [10]. The calculated pattern corresponds to the stress distribution at 1400 time steps. This case of rigid inclusions serves as the reference example for the comparison with the gratings composed by elastic scatterers.

4. Wave patterns due to elastic grating

The perfectly rigid grating is an idealization suited well for optics but hardly realized in solids. That is why we consider a more practical case with an elastic grating within an elastic matrix. To extend the