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A study of a cyclic viscoplasticity model based on hyperbolic sine form for the inelastic strain rate

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ABSTRACT

The viscoplastic hyperbolic sine form which describes the inelastic strain rate is combined with the Ohno-Wang kinematic hardening rule. Experiments under monotonic tensile loads at different strain rates as well as under cyclic loadings are used to obtain all model parameters with the trial and error method. The effects of strain jumps of the stress, the saturated visco-plasticity behavior under cyclic loadings and the stress relaxation for several metallic materials are simulated. The usage of the exponential form instead of the hyperbolic sine form for the inelastic strain rate is discussed finally.

1. Introduction

Nowadays many engineering components, which usually exhibit relaxation or creep behavior that can be regarded as rate-depended and eventually leads to ductile fracture, work at high temperature [1]. The demands for the safety design as well as the life prediction of the components are ever increasing. The importance for the construction on an appropriate constitutive model, which can accurately simulate the stress and strain of the materials that are made into engineering parts in such as nuclear plants, power generation industries, steam turbine and aerospace, is of particular significance. The rate dependent phenomenon is always correlated to the visco-plasticity of the material.

There have been many researches on the viscoplasticity model [2–8]. For Bodner's theory [2], a "directional hardening" was used, and the viscosity function combined a power and an exponential function. In the model proposed by Chaboche [3–5], several back stress terms were superposed to stand for the kinematic hardening, and the exponential form for the plastic strain rate was chosen. In the model introduced by Miller [6], there was only one back stress for kinematic hardening and a drag stress for isotropic hardening. In the Robinson model [7], only one term for the back stress was used to depict the static recovery terms, the drag stress was conducted to illustrate the isotropic hardening, the yield stress was used for the inelasticity behavior and the power function was employed for the viscoplastic flow description. The Krempl's VBO (viscoplasticity based on overstress) theory [8] that based on the overstress concept, formulated the back stress evolution with total strain rate instead of the viscoplastic strain rate. Since 1990s, ratcheting behavior has been extensively investigated by unified model, the total back stress was decomposed into several parts, the evolution of each part for the back stress was described by the Ohno-Wang model [9–12], and the plastic strain rate was always in the exponential form [13–16].

However, due to the complexity, ambiguity, and high cost, the unified model parameter determination is still a challenge for engineers, especially for the parameters of the viscoplasticity part. For Chaboche unified model, how to determine all the parameters, including kinematic part, visco-part, and isotropic part with strain memory, were discussed in detail in the literature [17]. Additionally, Tong et al. [18] optimized the visco-plastic parameters based on the initial values for the unified model. The parameters for the kinematic parts in the rate-independent model have been put forward by Jiang [19,20]. The method on how to determine the other parameters for the viscous part and the isotropic part can be found in references [13,21]. In the literatures, the visco-part parameters were obtained by the least square method and the transcendental equation solution, and parameters for the isotropic part were determined by the curve fitting for the maximum stress per cycle versus the accumulated plastic strain [21].

Little attention has been devoted to the viscous part which is usually used to describe the time or rate dependent behaviors in materials [22]. In this study, the hyperbolic sine function is adopted to depict the inelastic strain rate. It was reported that the hyperbolic sine form had been adopted in some literatures [6,10,23]. However, how to determine the related parameters for

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Nomenclature

A, B, D	visco-part parameters
b, Q	isotropic hardening parameters
D_{ijkl}	fourth order tensor for Hook elasticity
$\mathrm{d}p$	increment of the equivalent plastic strain
F_y	yield function
K_{ij}^k	normal direction for the kth part of back stress
k	yield stress
R	isotropic hardening stress
S_{ij}	partial stress tensor
α_{ij}	total back stress
$\alpha_{ij}^{(k)}$	the kth part of back stress
$\overline{\alpha^{(k)}}$	equivalent back stress
$\dot{\varepsilon}$	strain rate for tensile loading
ε_{ij}	total strain tensor
$\varepsilon_{ij}^e, \varepsilon_{ij}^p$	elastic and plastic strain tensor
$\dot{\varepsilon}_{ij}^p$	plastic strain rate
σ_{kl}	stress tensor
σ_v	visco-stress
$\xi_{ij}^{(k)}, m^{(k)}, r_{ij}^{(k)}$	kinematic part parameters

the form was not illustrated in detail. Additionally, the creep experiments which are required for the parameter determinations, are expensive and time-consuming. We approached a method to identify the rate-dependent parameters as well as the kinematic parameters from the stress–strain curve under the different strain/stress rates. Also the parameters for the isotropic hardening law were obtained from the cyclic experiment under the strain-controlled load. Finally, combined the hyperbolic sine function and the Ohno-Wang II model, we simulate the stress jumps for different strain rates, the cyclic visco-plasticity behaviors, and the stress relaxation for nickel-based alloy. The advantages for the hyperbolic sine function are discussed in the last section.

2. Viscoplasticity model

In the viscoplastic model, the total strain can be divided into two parts: the elastic strain ε_{ij}^e and the inelastic strain ε_{ij}^p that also can be regarded as the plastic strain.

$$\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^p \tag{1}$$

$$\varepsilon_{ij}^e = D_{ijkl}^{-1}\sigma_{kl} \tag{2}$$

D_{ijkl} is the fourth-order tensor of the Hook elasticity, σ_{kl} is the total stress tensor.

The hyperbolic sine function is chosen to describe the rate-dependent inelastic part,

$$\dot{\varepsilon}_{ij}^p = A\left[\sinh\left(\frac{F_y}{D}\right)\right]^B \frac{S_{ij}-\alpha_{ij}}{\|S_{ij}-\alpha_{ij}\|} \tag{3}$$

where A, B, and D are the parameters that are determined in the following section. S_{ij} and α_{ij} are the partial stress tensor and the back stress tensor, respectively. F_y is the yield function that be expressed as following,

$$F_y = \sqrt{1.5(S_{ij}-\alpha_{ij})(S_{ij}-\alpha_{ij})} - k - R \tag{4}$$

where k is the initial yield stress, R is the isotropic hardening stress which is used to describe the expansion or contraction of the hysteresis loop without consideration of the strain memory under the cyclic loading [18]. The evolution for the the isotropic hardening stress is,

$$\mathrm{d}R = b(Q-R)\mathrm{d}p \tag{5}$$

where b, Q are the isotropic hardening parameters. Q is the asymptotic value of the drag stress R that reaches the saturation, b represents the speed towards the saturation. $\mathrm{d}p$ is the increment of the equivalent plastic strain that can be illustrated by $\mathrm{d}p = \left((2/3)\mathrm{d}\varepsilon_{ij}^p : \mathrm{d}\varepsilon_{ij}^p\right)^{1/2}$. It is assumed that the initiation value for the isotropic internal variable R is zero.

The back stress α_{ij}that represents the kinematic hardening and indicates the Bauschinger effect due to plastic flow under cyclic loading, can be divided into several parts, and each part follows the Ohno-Wang II model [9,10,12]:

$$\dot{\alpha}_{ij}^{(k)} = \xi^{(k)}\left[\frac{2}{3}r^{(k)}\dot{\varepsilon}_{ij}^p - \left(\frac{\alpha^{(k)}}{r^{(k)}}\right)^{m^{(k)}}\left\langle \dot{\varepsilon}_{ij}^p : K_{ij}^{(k)}\right\rangle \alpha_{ij}^{(k)}\right] \tag{6}$$

where $\alpha_{ij}^{(k)}$ is the kth part of the back stress, and $\overline{\alpha^{(k)}} = \sqrt{1.5(\alpha_{ij}^{(k)} : \alpha_{ij}^{(k)})}$ is the equivalent back stress. $K_{ij}^k = \alpha_{ij}^{(k)}/\overline{\alpha^{(k)}}$denotes the normal direction for the kth part of back stress component$\alpha_{ij}^{(k)}$. The parameters $\xi_{ij}^{(k)}$, $m^{(k)}$and$r_{ij}^{(k)}$will be determined in the next sections. $\langle\ \rangle$is the Macauley bracket and satisfies$\langle x\rangle = (x+|x|)/2$.

3. Determination of the parameters

The data from uniaxial experiments in literatures is conducted to determine all the parameters except the isotropic part's in the viscoplastic model that is proposed in the previous section. It is noticed that all the tensors, such as stress σ_{ij}, back stress α_{ij}, total strain ε_{ij}, and plastic strain ε_{ij}^p, can be simplified as scalars under the uniaxial load.

The first step is to determine the parameters correlated to the rate-dependent inelastic part, i.e. the parameters A, B, and D in Eq. (3). Under the uniaxial tensile load, for a stress value large enough and more than the yield stress, it is found that the total stress σ can be divided into three parts: rate-dependent or viscous-stress σ_v, the kinematic part or back stress α, and the isotropic part $k$and R [18]:

$$\sigma = \alpha + R + (k+\sigma_v) \quad \text{for tensile load} \tag{7}$$

In the monotonic tensile experiments with different strain rates, at the same level of strain, the viscous-stress can be considered as the difference of the uniaxial stress at different strain rate. The reason is that the influence of the isotropic part R is negligible, and the back stress α can be considered to be the saturated value as the strain is large enough.

The experiments for the material of Ta-25W [24] under different strain rates of$10^{-5}/s$, $10^{-4}/s$, $10^{-3}/s$, and $10^{-2}/s$ are selected to illustrate how to determine the parameters correlated to the viscous part (Fig. 1). In the figure the results from simulations for the different strain rates are also plotted. As for the large strain, such as more than 15%in Fig. 1, the back stress can be regarded as saturated, and there's no distinction between the isotropic parts k and R for any different strain rate. From Eq. (7),